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MINIMAX ESTIMATION OF A BOUNDED BINOMIAL PARAMETER

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ABSTRACT

In this paper, we consider Minimax estimation of a Binomial parameter θ, when it is bounded above by a known constant. Some numerical results are derived.

KEYWORDS:

Minimaxity, Binomial distribution, bounded parameter, Balanced loss function.

1. INTRODUCTION

The problem considered in this paper is that of estimating the probability parameter θ of a binomial distribution under the additional assumption that it lies in a bounded interval of the form $[a,b] \neq [0,1]$, with $0 \leq a < b \leq 1$. An example of such a constraint occurring is in Warner (1965) random response model where the proportion is constrained to a symmetric interval $[a,1-a]$. Another example is given by a series system with two components C_1 and C_2 in which we are able to observe the proportion θ of successfully transmitted messages in a sample of size n. Suppose further that the reliability associated with C_1 is known and equal to b. Marchand and MacGibbon (2000) considered $\theta \leq b$ for quadratic and information-normalized loss functions.

In this article, we consider Minimax estimation of θ with $\theta \leq b$ under Zellner's (1994) Balanced loss function of the form

$$L(\theta,\delta(x)) = w(\frac{X}{n} - \delta(x))^2 + (1-w)(\theta - \delta(x))^2$$

Where w having a given value such that $0 < w < 1$.

2. NOTATION

Let X be a Binomial random variable $B(n,\theta)$, where $0 \leq \theta \leq b$, and (n, b) is known, with probability function

$$P_\theta(X=x)=\binom{n}{x}\theta^x(1-\theta)^{n-x} \quad , \; x\in\{0,1,2,\dots,n\} \; ; \; 0\le\theta\le b \tag{2.1}$$

For any estimator $\delta(x)$ we define the loss incurred by estimating θ with $\delta(x)$ to be

$$L(\theta,\delta(x))=w(\frac{x}{n}-\delta(x))^2+(1-w)(\theta-\delta(x))^2 \tag{2.2}$$

where w is a non-stochastic weight such that $0<w<1$.

The risk is defined as $R(\theta,\delta(x))=E_\theta[L(\theta,\delta(x))]$, and the Bayes risk with respect to the prior distribution $\tau(\theta)$ is

$$r(\tau,\delta)=E_\tau E_\theta(L(\theta,\delta(x))) \tag{2.3}$$

3. BAYES AND MINIMAX ESTIMATOR

For the prior satisfying $P(\theta=0)=1-P(\theta=b)=\varepsilon$, the corresponding Bayes rule is given by

$$\delta_\varepsilon(x)=[b(1-w)(1-A)]I_{[x=0]}+\left[w\frac{x}{n}+(1-w)b\right]I_{[x>0]} \tag{3.1}$$

with $A=\dfrac{\varepsilon}{\varepsilon+(1-\varepsilon)(1-b)^n}$, and associated risk

$$\begin{aligned}R(\theta,\delta_\varepsilon)&=wE(\frac{X}{n}-\delta)^2+(1-w)E(\theta-\delta)^2\\&=(1-\theta)^n(1-w)^2Ab[Ab-2(b-\theta)]\\&\quad+\frac{w(1-w)}{n}\theta(1-\theta)+(b-\theta)^2(1-w)^2\end{aligned} \tag{3.2}$$

Then the rule δ_ε will be Minimax if

i) $R(0,\delta_\varepsilon)=R(b,\delta_\varepsilon)$, and
ii) $R(\theta,\delta_\varepsilon)\le R(0,\delta_\varepsilon)$ for all $\theta\in[0,b]$.

Condition (i) is equivalent to

$$(1-w)^2b^2(1-A)^2=(1-b)^n(1-w)^2A^2b^2+\frac{w(1-w)}{n}b(1-b)$$

$$\varepsilon=\varepsilon^*=\frac{(1-b)^n}{[1-(1-b)^n]}\left[\sqrt{\frac{n(1-w)b}{c}}-1\right] \quad \text{and} \quad A=A^*=\frac{1-\sqrt{\dfrac{c}{n(1-w)b}}}{1-(1-b)^n}$$

where

$$c = n(1-w)b(1-b)^n + w(1-b)(1-(1-b)^n).$$

Since ε^* must be positive then $b \geq \dfrac{w}{w+n(1-w)} = b_*(n)$

For n=1 we have

$$\frac{\partial R(\theta,\delta_{\varepsilon^*})}{\partial\theta} = (1-w)^2 A^* b\left[-A^* b + 2b + 2 - 4\theta\right] + w(1-w)(1-2\theta) + 2(1-w)^2(\theta - b)$$

and

$$\frac{\partial^2 R(\theta,\delta_{\varepsilon^*})}{\partial\theta^2} = -4(1-w)^2 A^* b - 2w(1-w) + 2(1-w)^2$$

for condition (ii) to be satisfied, it is necessary that

$$\frac{\partial^2 R(\theta,\delta_{\varepsilon^*})}{\partial\theta^2} \geq 0$$

which implies that

$$b \leq \frac{3-4w}{4-4w} = b^*(n) \qquad \text{for } n=1 \tag{3.3}$$

and for $n \geq 2$ we have

$$\frac{\partial R(\theta,\delta_{\varepsilon^*})}{\partial\theta} = (1-\theta)^{n-1}(1-w)^2 A^* b\left[-nA^* b + 2nb + 2 - 2\theta - 2n\theta\right] + \frac{w(1-w)}{n}(1-2\theta) + 2(\theta-b)(1-w)^2$$

and

$$\frac{\partial^2 R(\theta,\delta_{\varepsilon^*})}{\partial\theta^2} = nA^* b(1-\theta)^{n-2}(1-w)^2\left[-(n-1)b(2-A^*) - 4 + 2\theta(n+1)\right] + 2(1-w)\left[1 - w - \frac{w}{n}\right]$$

Since the second derivative has at most one sign change on $[0,b]$ from $-$ to $+$ condition (ii) is equivalent to

$$\frac{\partial R(\theta,\delta_{\varepsilon^*})}{\partial\theta}\Big|_{\theta=0} = (1-w)^2 A^* b(-nA^* b + 2nb + 2) + \frac{w(1-w)}{n} - 2b(1-w)^2 \leq 0$$

and this imply that, $b \leq b^*(n)$ where $b^*(n)$ is the unique root of the equation

$$(1-w)^2A^*b(-nA^*b+2nb+2)+\frac{w(1-w)}{n}-2b(1-w)^2=0 \text{ on } [0,1]$$

Theorem 3.1. δ_{ε^*} is Minimax if $b_*(n)<b<b^*(n)$. When this is the case, the two points prior such that $P(\theta=0)=1-P(\theta=b)=\varepsilon^*$ is the least favorable prior and the Minimax risk is given by $R(0,\delta_{\varepsilon^*})=(1-A^*)^2b^2(1-w)^2$.

Some values of $b_*(n)$ and $b^*(n)$ are presented in table 1. It must be noted that for some cells which these values cannot be estimated we put *.

Table 1: Values of $b_*(n)$ and $b^*(n)$ for different values of n and w

	w =.05	w =.10	w =.15	w =.20	w =.25
n=1	.050-.737	.100-.722	.150-.706	.200-.688	.250-.667
n=2	.026-.428	.053-.431	.081-.430	.111-.425	.143-.411
n=3	.017-.298	.036-.305	.055-.309	.077-.311	.100-.309
n=4	.013-.228	.027-.235	.042-.241	.059-.245	.077-.247
n=5	.010-.185	.022-.191	.034-.197	.048-.202	.063-.205
n=6	.009-.155	.018-.161	.029-.167	.040-.171	.053-.175
n=7	.007-.134	.016-.139	.025-.144	.034-.149	.045-.152
n=8	.007-.117	.014-.122	.022-.127	.030-.131	.040-.135
n=9	.006-.105	.012-.109	.019-.114	.027-.118	.036-.121
n=10	.005-.095	.011-.099	.017-.103	.024-.107	.032-.110
n=11	.005-.086	.010-.090	.016-.094	.022-.097	.029-.101
n=12	.004-.079	.009-.083	.014-.086	.020-.090	.027-.093
n=13	.004-.073	.008-.077	.013-.080	.019-.083	.025-.086
n=14	.004-.068	.008-.071	.012-.074	.018-.077	.023-.080
n=15	.003-.063	.007-.067	.012-.070	.016-.072	.022-.075
n=16	.003-.060	.007-.063	.011-.065	.015-.068	.020-.071
n=17	.003-.056	.006-.059	.010-.062	.014-.064	.019-.067
n=18	.003-.053	.006-.056	.010-.058	.014-.061	.018-.063
n=19	.003-.050	.006-.053	.009-.055	.013-.058	.017-.060
n=20	.003-.048	.006-.050	.009-.053	.012-.055	.016-.057
n=21	.003-.046	.005-.048	.008-.050	.012-.052	.016-.054
n=22	.002-.043	.005-.046	.008-.048	.011-.050	.015-.052
n=23	.002-.042	.005-.044	.008-.046	.011-.048	.014-.050
n=24	.002-.040	.005-.042	.007-.044	.010-.046	.014-.048
n=25	.002-.038	.004-.040	.007-.042	.010-.044	.013-.046
n=26	.002-.037	.004-.039	.007-.041	.010-.042	.013-.044
n=27	.002-.036	.004-.037	.006-.039	.009-.041	.012-.043
n=28	.002-.034	.004-.036	.006-.038	.009-.039	.012-.041
n=29	.002-.033	.004-.035	.006-.037	.009-.038	.011-.040
n=30	.002-.032	.004-.034	.006-.035	.008-.037	.011-.038

Table 1: Continued

	w =.30	w =.35	w =.40	w =.45	w =.50
n=1	.300-.643	.350-.625	.400-.583	.450-.545	*
n=2	.176-.383	.212-.313	*	*	*
n=3	.125-.301	.152-.278	*	*	*
n=4	.097-.244	.119-.234	*	*	*
n=5	.079-.205	.097-.200	*	*	*
n=6	.067-.176	.082-.173	*	*	*
n=7	.058-.154	.071-.153	*	*	*
n=8	.051-.137	.063-.137	*	*	*
n=9	.045-.123	.056-.124	*	*	*
n=10	.041-.112	.051-.113	*	*	*
n=11	.038-.103	.047-.104	*	*	*
n=12	.034-.095	.043-.096	*	*	*
n=13	.032-.088	.040-.089	*	*	*
n=14	.030-.082	.037-.083	*	*	*
n=15	.028-.077	.035-.078	*	*	*
n=16	.026-.073	.033-.074	*	*	*
n=17	.025-.069	.031-.070	*	*	*
n=18	.023-.065	.029-.066	*	*	*
n=19	.022-.062	.028-.063	*	*	*
n=20	.021-.059	.026-.060	*	*	*
n=21	.020-.056	.025-.057	*	*	*
n=22	.019-.054	.024-.055	*	*	*
n=23	.018-.051	.023-.053	*	*	*
n=24	.018-.049	.022-050	*	*	*
n=25	.017-.047	.021-.049	*	*	*
n=26	.016-.046	.020-.047	*	*	*
n=27	.016-.044	.020-.045	*	*	*
n=28	.015-.043	.019-.044	*	*	*
n=29	.015-.041	.018-.042	*	*	*
n=30	.014-.040	018-.041	*	*	*

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