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# Stochastic Models of Computer Communication Systems

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SUMMARY

This paper describes some examples of the stochastic models found useful in the design and analysis of advanced computer and communication systems. Our major theme might be termed the control of contention. As illustrations of this theme we discuss concurrency control procedures for databases, dynamic channel assignment for cellular radio, and random access schemes for the control of a broadcast channel. We emphasize asymptotic properties of product-form distributions and we present some new results on the stability of acknowledgement based random access schemes.

*Keywords*: CONCURRENCY CONTROL; DATABASE LOCKING; DYNAMIC CHANNEL ASSIGNMENT; CELLULAR RADIO; RANDOM ACCESS SCHEMES; PRODUCT-FORM DISTRIBUTIONS

## 1. INTRODUCTION

This paper is intended to describe to the Society some examples of the stochastic models found useful in the design and analysis of advanced computer and communication systems. The examples chosen are broadly concerned with what might be termed the *control of contention*, and an attempt has been made to provide enough of the technical background to motivate the models considered.

In Section 2 we describe a probabilistic model, due to Mitra (1985), for conflicts among transactions in a database. Such conflicts can arise in distributed computer systems, where to ensure the consistency of a database it is often necessary to forbid the concurrent execution of transactions involving common items: a transaction must then contend with other transactions for access to the items it requires. Mitra (1985) has shown that his model can be used to answer some important design questions concerning concurrency control procedures which use exclusive and non-exclusive locks. Mitra's results are based upon a product-form solution; we indicate how his asymptotic formulae can be extended beyond the range of light traffic and the assumption of an unstructured database.

In Section 3 we discuss one of the many interesting problems which arise in connection with cellular radio. Cellular radio makes efficient use of a limited number of radio channels by allowing the repeated reuse of each channel in sufficiently separated spatial regions. The topic we consider is contention between different regions for the use of dynamically assigned channels. Everitt and Macfadyen (1983) have described an analytically tractable method of dynamic channel assignment, which they term the maximum packing strategy. Again a product form solution is involved: from this it is easy to obtain asymptotic formulae applicable in light traffic. These formulae establish the advantage of the strategy over a fixed channel assignment for low enough loss probabilities, but the advantage disappears as traffic and the number of channels increase. The real potential of dynamic schemes is their ability to cope automatically with traffic intensities which fluctuate in space and time.

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We go to some lengths in Sections 2 and 3 to indicate relationships with work on random fields (Preston, 1976).

In Section 4 we discuss random access schemes for the control of a broadcast channel shared by a large number of users. This is an example of an increasingly important class of problem: high data rates and extensive parallel processing lead to distributed computer systems whose performance is often constrained by fundamental limitations (imposed by the speed of light) on how well the various components can coordinate their activity. We outline the results of Mikhailov (1979) and Hajek (1982a) on random access schemes using channel feedback, and we present some new results on the stability of acknowledgement based random access schemes.

Queueing network models have been used extensively in connection with packet-switching communication networks and time-shared multiprogrammed computer facilities. See Kleinrock (1976) for an introduction to this area, including an account of the role of models in the design of the ARPANET system, and Kelly (1979) and Gelenbe and Mitrani (1980) for further details of the models themselves. A major aim of these models has been to relate performance measures such as queue lengths, throughput and delays to the dimensioning of system components. The models considered in this paper have a rather different emphasis, their main aim being to provide insight into the mechanisms by which contention limits the performance of a system.

No attempt is made here to survey all the important probabilistic questions concerning the systems described: for a fuller perspective the reader is referred to Christodoulakis (1984) on database systems, Davis (1984) on cellular radio and Stuck (1984) on channel access methods. An indication of the enormous scope for probabilistic modelling in the design and analysis of computer communication systems can be found in the volumes edited by Disney and Ott (1982), Baccelli and Fayolle (1984), Iazeolla *et al.* (1984), Gelenbe (1984) and Gopinath (1985).

## 2. CONCURRENCY AND DATABASE LOCKING

When a large database is accessed by many different processors some form of concurrency control is usually necessary to ensure that the database remains consistent. For example, suppose that transactions 1 and 2 of Fig. 1 involve a database containing items $X_A$, $X_B$, and $X_C$. Transaction 1 is intended to transfer an amount $K_{AB}$ from the account of $A$ to the account of $B$, and transaction 2 an amount $K_{BC}$ from $B$ to $C$. If the transactions are executed concurrently on

| *Transaction 1* | | *Transaction 2* | |
|---|---|---|---|
| 1.1 | Load $X_A$ | 2.1 | Load $X_B$ |
| 1.2 | Subtract $K_{AB}$ | 2.2 | Subtract $K_{BC}$ |
| 1.3 | Store $X_A$ | 2.3 | Store $X_B$ |
| 1.4 | Load $X_B$ | 2.4 | Load $X_C$ |
| 1.5 | Add $K_{AB}$ | 2.5 | Add $K_{BC}$ |
| 1.6 | Store $X_B$ | 2.6 | Store $X_C$ |

Fig. 1. The need for concurrency control

parallel processors they may not have the intended effect: for example the sequence of operations 1.1-1.5, 2.1, 1.6, 2.2-2.6 leaves the account of $B$ deficient by an amount $K_{AB}$. Database locking aims to prevent this by placing restrictions upon which transactions can be processed concurrently. Locking, however, impairs efficiency and it is important to be able to assess the performance of different locking strategies.

We describe now a model from the class introduced by Mitra and Weinberger (1984) and Mitra (1985). Let the database consist of $N$ items. Associate with each transaction a list of distinct items. The list is partitioned into two sets, with items in the leading set requiring exclusive locks and items in the trailing set requiring only non-exclusive locks. (Usually to read an item requires a non-exclusive lock, while to write to an item requires an exclusive lock. The formal distinction between the two types of lock will be made later, in equation (2.1).) Requests for transaction processing arrive exogeneously to the database. On arrival of a request the database lock manager decides to either grant or refuse the locks required on the following basis. Let $W_d$ and $R_d$ be the sets of exclusively locked and non-exclusively locked items, respectively, in the database at the time of arrival. Let $W_a$ and $R_a$ be the sets of items required to be exclusively locked and non-exclusively locked, respectively, by the arriving transaction. The locks are granted if

$$(W_a \cap W_d) \cup (W_a \cap R_d) \cup (R_a \cap W_d) = \Phi \tag{2.1}$$

and denied otherwise. Once granted locks are not released until the entire processing of the transaction is complete, and accepted transactions are processed in parallel. If locks are denied then the request is blocked. Blocked requests are discarded (perhaps to try again at a later time—we return to this point later). An important question is whether it is worthwhile classifying locks as exclusive or non-exclusive, since the operation of a database lock manager is much simpler when all locks are treated as exclusive.

Assume that the arrival stream of requests is Poisson at rate $\nu$, and that the processing times for transactions are independent random variables, arbitrarily distributed with unit mean. Assume that a request requires $w$ items to be exclusively locked and $r$ items to be non-exclusively locked, and that each of the

$$\binom{N}{r+w}\binom{r+w}{w}$$

possible choices for these items is equally likely. Define the state of the system to be the set of transactions $S = \{T_1, T_2, \ldots, T_n\}$ undergoing processing, where a transaction $T_i$ is defined by its locks, $T_i = (W_i; R_i)$, and $n = |S|$ is the concurrency of the system. A state $S$ is *admissible* if the exclusively locked items in the constituent transactions are mutually disjoint and the union of all exclusively locked items is disjoint from the union of all non-exclusively locked items. Let $\Omega$ be the collection of admissible states. Since processing times are arbitrarily distributed the stochastic process describing the evolution of the state $S$ is not Markov; nevertheless the stationary distribution for $S$ takes the straightforward form

$$\pi(S) = B\rho^{|S|} \qquad S \in \Omega$$

where

$$\rho = \nu \left\{ \binom{N}{r+w} \binom{r+w}{w} \right\}^{-1}$$

and $B$ is a normalizing constant, chosen to ensure the distribution sums to unity (see, for example: Kelly, 1979, Theorem 3.14; Burman *et al.*, 1984). To obtain, for example, the distribution of the concurrency, $\pi(S)$ must be summed over the appropriate subsets of $\Omega$. Thus

$$P\{|S| = n\} = B\rho^n \frac{N!}{(w!)^n n!\,(N-nw)!} \binom{N-nw}{r}^n, \tag{2.2}$$

after counting the various ways in which $n$ transactions can be concurrently processed. Suppose now we let $N \to \infty$ holding $\nu$, $r$ and $w$ fixed. Under this regime, which might be termed light traffic, a Poisson limit emerges from the form (2.2):

$$P\{|S| = n\} \to e^{-\nu} \frac{\nu^n}{n!}. \tag{2.3}$$

It can also be deduced that the probability that an arriving request is blocked is

$$\frac{\nu}{N}(w^2 + 2wr) + o\left(\frac{1}{N}\right), \tag{2.4}$$

a special case of the result of Mitra (1985), who allows $w$ and $r$ to vary over transactions. The factor $w^2 + 2wr$ has a natural interpretation in terms of the probability that two transactions $T_1 = (W_a; R_a)$ and $T_2 = (W_d; R_d)$ violate condition (2.1), and helps quantify the benefit of non-exclusive locks.

The asymptotic estimate (2.4) provides good approximations for moderate values of $N$ provided the blocking probability is small. More widely valid, although more complex, approximations can be developed by considering the moderate traffic limiting regime, in which $N \to \infty$ with $\nu \sim \lambda N$, holding $\lambda$, $r$ and $w$ fixed. Then, for any $\epsilon > 0$,

$$P\left\{ \frac{w}{N} |S| - p \quad > \epsilon \right\} \to 0$$

where $p$ is the unique solution in the interval (0, 1) of the equation

$$p = \lambda w(1-p)^{w+r} \exp\left(\frac{-pr}{1-p}\right).$$

To prove this result requires a rather close analysis of the expression (2.2), using Stirling's formula to estimate the factorials. In the limit $p$ is the proportion of items in the database which are exclusively locked,

$$q = (1-p)\left\{1 - \exp\left(\frac{-pr}{(1-p)\,w}\right)\right\}$$

is the proportion which are non-exclusively locked, and $(1-p-q)^w(1-p)^r$ is the probability an arriving request can be accepted.

The model discussed is just one from a rich class which can be analysed explicitly. As described it is essentially an infinite server queue with a particular form of arrival process. The infinite server queue can be replaced by any quasi-reversible network (for an example see Mitra and Weinberger, 1984) and the arrival process can be generalized. The assumption that blocked requests are discarded is, however, important. In practice blocked requests may be resubmitted for processing. If the delay before resubmission is large enough the model will be adequate, with the arrival rate $\nu$ adjusted to achieve a specified throughput. Morris and Wong (1984) study a model which allows queueing, and Kelly (1986a) compares queueing and loss systems with the same throughput.

The model as so far described assumes no structure on the database, in the sense that every possible choice for the items required by a transaction is equally likely. How might the conclusions be affected if the items required by a transaction are "close" to one another? Suppose for example that items form the set $\{(i,j): i,j = 1, 2, \ldots, M\}$, and that a transaction requires an exclusive lock on a single randomly chosen item from the set $\{(i,j): i,j = 2, 3, \ldots, M-1\}$ and non-exclusive locks on the adjacent four items. Requests arrive at a fixed overall rate $\nu$. Then as $M^2 = N \to \infty$ the Poisson limit (2.3) again emerges, but the probability an arriving request is blocked is now

$$\frac{5\nu}{N} + o\left(\frac{1}{N}\right).$$

The factor 5 is smaller than the factor 9 suggested by the estimate (2.4) because of the constraints now placed on the locks a transaction can require (observe that for a request arriving at an internal site there are just 5 possible transactions which could block it). Similar light traffic results can be obtained for other graph-theoretic representations of the proximity of items. The case of moderate traffic is however not as straightforward, as we can illustrate with a model already familiar from the study of phase transitions (Spitzer, 1975; Preston, 1976; Kinderman and Snell, 1980; Zachary, 1983).

Let $A$ be the infinite tree with $r$ edges emanating from each vertex and let $A_M$ be the finite subgraph consisting of a distinguished vertex $O$ and each vertex not more than $M$ steps from it. Thus in the tree $A_M$ each vertex has either one or $r$ edges emanating from it. Call these vertices external and internal respectively. Associate an item with each vertex of $A_M$ and suppose there are two types of transaction. A transaction may require an exclusive lock on one internal vertex and non-exclusive locks on the $r$ neighbouring vertices, such requests arriving at rate $\lambda$ for each internal vertex; or a transaction may require an exclusive lock on one external vertex and a non-exclusive lock on the neighbouring internal vertex, such requests arriving at rate $\lambda_e$ for each external vertex. Let $x_v = 1$ or 0 according as vertex $v$ is under an exclusive lock or not. Then an admissible state is described by a collection $\mathbf{x}_M = (x_v, v \in A)$ with the property that $x_i x_j = 0$ if $i$ and $j$ are neighbours. The stationary distribution of $\mathbf{x}_M$ can be constructed as follows. Let $s(v)$ be the minimum number of steps from vertex $v$ to an external vertex and let $u(v)$ be the neighbour of $v$ which is one step nearer to vertex $O$ than $v$. Let $P_m = (P_m(i,j); i, j = 0, 1)$ be the stochastic matrix defined by

$$P_m = \begin{pmatrix} a_m & 1 - a_m \\ 1 & 0 \end{pmatrix}$$

for $a_1, a_2, \ldots, a_m$ to be determined, and set

$$\pi(\mathbf{x}_M) = \pi(x_0) \prod_{v \neq x_0} P_{s(u(v))}(x_{u(v)}, x_v)$$

for $\pi(x_0)$ to be determined. Under this distribution the values observed along a path of length $M$ from vertex $O$ to an external vertex are generated by a non-homogeneous Markov chain, with transition matrices $P_M, P_{M-1}, \ldots, P_1$. Let $\pi_m$ be the induced probability that $x_v = 1$ for a vertex $v$ with $s(v) = m$. Then $\pi_M = \pi(x_0)$, and

$$\pi_{m-1} = (1 - \pi_m)(1 - a_m) \qquad (m = 1, 2, \ldots, M) \tag{2.5}$$

For $\pi(\mathbf{x}_M)$ to be the stationary distribution of $\mathbf{x}_M$ the following detailed balance conditions must be satisfied

$$(1 - \pi_1) a_1 \lambda_e = (1 - \pi_1)(1 - a_1) \tag{2.6}$$

$$(1 - \pi_m) a_m a_{m-1}^{r-1} \lambda = (1 - \pi_m)(1 - a_m), \qquad (m = 2, 3, \ldots, M), \tag{2.7}$$

$$(1 - \pi_M) a_M^r \lambda = \pi_M. \tag{2.8}$$

Equation (2.8), for example, arises by considering an arrival at, or departure from, vertex $O$ of a transaction centred there. Equation (2.6) determines $a_1$ in terms of $\lambda_e$. Equation (2.7) becomes

$$a_m = \frac{1}{1 + \lambda a_{m-1}^{r-1}} \qquad (m = 2, 3, \ldots, M), \tag{2.9}$$

determining $a_2, a_3, \ldots, a_M$. The probability $\pi_M$ is then given by equation (2.8), and $\pi_0, \pi_1, \ldots, \pi_{M-1}$ by the recursion (2.5).

The recursion (2.9), illustrated in Fig. 2, has one fixed point, $\bar{a}$, the positive root of $\bar{a} + \lambda \bar{a}^r = 1$. There is an associated value $\bar{\lambda}_e = \bar{a}^{-1} - 1$ which generates a solution $a_1 = a_2 = \ldots = a_M = \bar{a}$, and $\pi_0 = \pi_1 = \ldots = \pi_M = (2 - \bar{a})^{-1}$. This is an appealing solution, since under it the stationary distribution over a vertex and its neighbours is identical at each internal vertex. For example the probability of acceptance of a request centred at any internal vertex is $\bar{a}^r (2 - \bar{a})^{-1}$. However if

$$\lambda > \frac{1}{r-1} \left( \frac{r-1}{r-2} \right)^r$$

then $f'(\bar{a}) < -1$ and so the fixed point $\bar{a}$ is unstable: a value of $\lambda_e$ arbitrarily close to $\bar{\lambda}_e$ gives rise to a sequence $a_1, a_2, \ldots, a_M$ which oscillates away from $\bar{a}$. The stationary distribution over a vertex $v$ and its neighbours, and in particular the derived blocking probability, will then depend upon the location of $v$, and markedly upon whether $s(v)$ is even or odd.

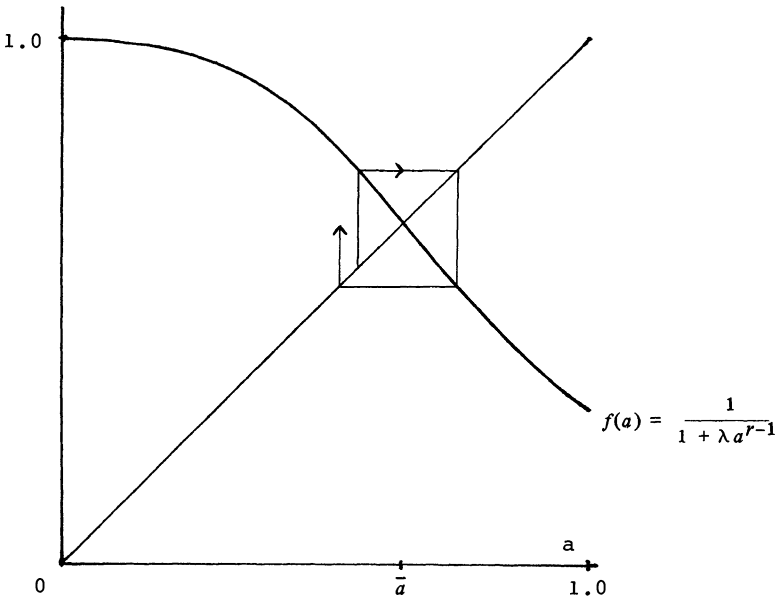


Fig. 2. Instability of the fixed point

For $r > 2$ most of the vertices of the tree $A_M$ are external vertices, and so the above behaviour is perhaps not unexpected. However, related phenomena occur when the underlying graph is a section of the two dimensional lattice, the model then resembling the Ising model of an anti-ferromagnet. Of course these must be regarded as highly idealized graphical structures, but the mere possibility of such phenomena is worth noting. Simulations of such systems may well exhibit the features associated with phase transition: long periods in nearly stationary regimes with sudden switches between these regimes, and marked hysteresis effects (cf. Kinderman and Snell, 1980). See Kelbert and Suhov (1983), Suhov (1984) for some theoretical work on phase transitions in communication networks; Nelson (1984) for an approach through catastrophe theory; and Akinpelu (1983) for a description of the hysteresis effects uncovered by simulation of a model of a telephone network.

## 3. DYNAMIC CHANNEL ASSIGNMENT FOR CELLULAR RADIO

In the near future there is expected to be a large growth in mobile telephony, made possible by the introduction of cellular radio (Appleby, 1983). The idea of cellular radio is that the coverage area is divided into cells, each with its own radio base station. These cells are then formed into clusters (three cells per cluster in the example illustrated in Fig. 3). The limited number of available radio channels are divided equally in a fixed pattern between the cells in a cluster, and the pattern repeated to fill the coverage area. In this way the same radio channel may be used in different cells, separated by a distance sufficient to limit interference. When a mobile user is involved in a call she is connected via the base station of her cell into the telephone network. Under a *fixed channel assignment* a call attempt is lost if all the channels allocated to the cell concerned are already in use. Observe, though, that the cell might be able to accommodate an

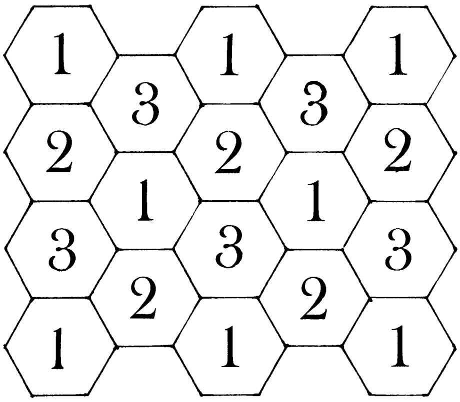


Fig. 3. A cellular radio layout

additional call if it could borrow a channel, allocated to an adjacent cell, provided this use of the channel did not interfere with any calls currently in progress. This observation has led to interest in methods of *dynamic channel assignment*, and it is important to know when such methods provide an improvement over a fixed channel assignment.

We shall model a fixed channel assignment by the standard Erlang scheme. We assume cells are independent, call attempts arrive at a cell in a Poisson stream of rate $\nu$, call lengths are arbitrarily distributed with unit mean, and blocked calls are lost. Then if $c$ channels are allocated to a cell the loss probability (the equilibrium probability that an arriving call is lost) is given by Erlang's formula

$$L_{\text{FCA}} = \frac{\nu^c}{c!}\left[\sum_{n=0}^{c} \frac{\nu^n}{n!}\right]^{-1}.$$

It is worth noting two features of cellular radio not represented in this simple model. When a mobile engaged on a call moves from one cell to another the mobile is "handed-off" from the cell it is leaving to a new radio channel in the cell it is approaching (this happens without the user noticing). We ignore hand-off, and we also ignore mobile-to-mobile calls.

The method of dynamic channel assignment we shall describe was introduced by Everitt and Macfadyen (1983). Construct a graph $A$ as follows: let $A$ have a vertex for each cell of the system, and let $A$ have an edge joining vertices $u$ and $v$ if a channel cannot be used simultaneously in cells $u$ and $v$. Let $n_v$ be the number of calls in progress in cell $v$ and let $\mathrm{n} = (n_v, v \in A)$. Call a state $\mathrm{n}$ *admissible* if there exists an allocation of channels to calls such that any given channel is not in use at both ends of an edge of $A$. The *maximum packing strategy* accepts a call whenever this leads to a state $\mathrm{n}$ which is admissible. The strategy thus accepts a call whenever possible, even if this involves a rearrangement of the channels allocated to calls already in progress. If $\Omega$ is the set of admissible states then the stationary distribution for $\mathrm{n}$ is

$$\pi(\mathrm{n}) = B \prod_{v \in A} \frac{\nu_v^{n_v}}{n_v!}, \quad \mathrm{n} \in \Omega. \tag{3.1}$$

Everitt and Macfadyen (1983) give an illuminating discussion of the relationship between the set $\Omega$ and the colourability properties of the graph $A$. The normalizing constant $B$, and hence loss probabilities, are, in general, difficult to obtain. Next we shall consider in detail a special case.

Suppose that cells are located at positions $v = -N, -N+1, \ldots, N-1, N$, that the arrival rate is $\nu$ at each cell, that a total of $C$ channels are available, and that interference prevents a channel's use in two adjacent cells. Then a state $\mathsf{n}$ is admissible provided

$$n_v + n_{v+1} \leqslant C \quad (v = -N, -N+1, \ldots, N-1).$$

From expression (3.1) it follows that the loss probability at an internal cell is

$$L_{\mathrm{MP}} = B\left[\frac{\nu^C}{C!} + 2\sum_{n=0}^{C-1} \frac{\nu^n}{n!}\cdot\frac{\nu^{C-n}}{(C-n)!} + o(\nu^C)\right]$$

$$= B\left[\frac{2(2\nu)^C}{C!} - \frac{\nu^C}{C!} + o(\nu^C)\right]$$

as $\nu \to 0$. But $B = \{1 + O(\nu)\}^{-1}$ and so

$$L_{\mathrm{MP}} \sim \frac{2^{C+1}-1}{C!}\,\nu^C \quad \text{as} \quad \nu \to 0. \tag{3.2}$$

If under a fixed channel assignment the cell had been allocated $c = C/2$ channels (assume for this purpose that $c$ is integral) then from (3.1)

$$L_{\mathrm{FCA}} \sim \{(\tfrac{1}{2}C)!\}^{-1}\,\nu^{C/2} \quad \text{as} \quad \nu \to 0. \tag{3.3}$$

This argument extends to more general cell structures (for that of Fig. 3 it shows

$$L_{\mathrm{MP}} \sim \frac{6(3^C - 2^C)+1}{C!}\,\nu^C, \quad L_{\mathrm{FCA}} \sim \{(\tfrac{1}{3}C)!\}^{-1}\,\nu^{C/3} \quad \text{as} \quad \nu \to 0)$$

and indicates that dynamic channel assignment will have an advantage over fixed channel assignment for small enough values of $\nu$.

To proceed further we look in more detail at the distribution $\pi(\mathsf{n})$. Just as in Section 2 we can rewrite this distribution in an alternative form (Brook, 1964; Spitzer, 1971, Preston, 1976; observe that the special case $C = 1$ of the model considered here corresponds to the special case $r = 2$ of the final model of Section 2). Let $Q = (Q(n,m); n, m = 0, 1, \ldots, C)$ be defined by

$$Q(n,m) = \frac{\nu^m}{m!} \quad (n = 0, 1, \ldots, C-m)$$

$$= 0 \quad (n = C-m+1, \ldots, C)$$

and define $Q^u(n,m)$ by matrix multiplication. Let

$$P_u(n,m) = \frac{Q(n,m)\,\Sigma_{k=0}^{C}\,Q^{u-1}(m,k)}{\Sigma_{k=0}^{C}\,Q^u(n,k)},$$

and observe that $P_u$ is a stochastic matrix. Then the stationary distribution $\pi(\mathsf{n})$ can be written in the form

$$\pi(\mathsf{n}) = \pi_N(n_0)\prod_{v=0}^{N-1} P_{N-v}(n_v, n_{v+1})\,P_{N-v}(n_{-v}, n_{-v-1})$$

where the probability distribution $(\pi_N(n_0), n_0 = 0, 1, \ldots, C)$ is determined by

$$\pi_N(n_0+1)=\pi_N(n_0)\,\frac{\nu}{n_0+1}\left[\frac{\Sigma_{k=0}^{C}\,Q^N(n_0+1,k)}{\Sigma_{k=0}^{C}\,Q^N(n_0,k)}\right]^2 \qquad (n_0=0,1,\ldots,C-1)$$

Perhaps the simplest way to check this is to verify that the expression for $\pi(\mathsf{n})$ satisfies the detailed balance conditions. Thus in equilibrium $\mathsf{n}$ has the distribution of a non-homogeneous Markov chain. Suppose now we let $N\to\infty$. Since $Q$ is a primitive non-negative matrix,

$$\frac{1}{a^u}\;Q^u(n,m)\to r(n)\,l(m) \quad \text{as} \quad u\to\infty$$

where $a$ is the largest eigenvalue of $Q$, and $r$, $l$ are its positive right and left eigenvectors normalized so that $l^{\mathrm{T}}r=1$ (Seneta, 1980). Thus, as $N\to\infty$, $P_{N-v}$ and $\pi_N$ tend to limits $P$ and $\pi$ given by

$$P(n,m)=\frac{Q(n,m)\,r(m)}{ar(n)}$$

and

$$\pi(n)=b\,\frac{\nu^n}{n!}\,r(n)^2,$$

where $b$ is a normalizing constant, chosen so that $(\pi(n), n=0,1,\ldots,C)$ is a probability distribution. Note that $(\pi(n), n=0,1,\ldots,C)$ satisfies the detailed balance conditions for the stochastic matrix $P$. Thus in the limit $(n_{-s}, n_{-s+1},\ldots,n_s)$ is distributed as a fragment from the sample path of the stationary, homogeneous, reversible Markov chain determined by $P$, for any finite $s$. The corresponding loss probability is

$$L_{\mathrm{MP}}=\sum_{n=0}^{C}\pi(n)\,[1-\{1-P(n,C-n)\}^2] \tag{3.4}$$

$$=1-b\sum_{n=0}^{C-1}\frac{\nu^n}{n!}\,r(n+1)^2.$$

The second equality can be checked algebraically, but the latter expression arises naturally from the observation that the carried traffic per cell is $\Sigma n\pi(n)$. As $\nu\to 0$ the eigenvalue $a\to 1$ and we can choose the eigenvector $r$ so that $r(n)\to 1$, $n=0,1,\ldots,C$. This implies that $L_{\mathrm{MP}}$ as given by (3.4) satisfies (3.2): it is reassuring that this relation has remained intact under the limiting operation $N\to\infty$.

Figure 4 illustrates loss probabilities for the case $C=2$. Then

$$L_{\mathrm{FCA}}=\frac{\nu}{1+\nu},\quad L_{\mathrm{MP}}=\frac{p^2(14-10p-5p^2+3p^3)}{2(2+p^2-2p^3)}$$

where $p=P(1,1)=\nu/a$ is the unique solution in (0, 1) of the equation

$$\nu(1-p)\,(2-p^2)=2p.$$

For $\nu$ near zero the form of these curves is given by relations (3.2) and (3.3). But as $\nu$ increases $L_{\mathrm{MP}}$ rises to $L_{\mathrm{FCA}}$, and indeed $L_{\mathrm{MP}}$ is marginally greater than $L_{\mathrm{FCA}}$ if $\nu>2.6$. At a high enough traffic intensity a fixed channel assignment loses fewer calls than the maximum packing strategy. This is perhaps surprising, since in any given state if the maximum packing strategy rejects a call then so must any other strategy. A heuristic explanation is that the maximum packing strategy disrupts the close spatial packing of channels achieved by a fixed channel assignment. Another

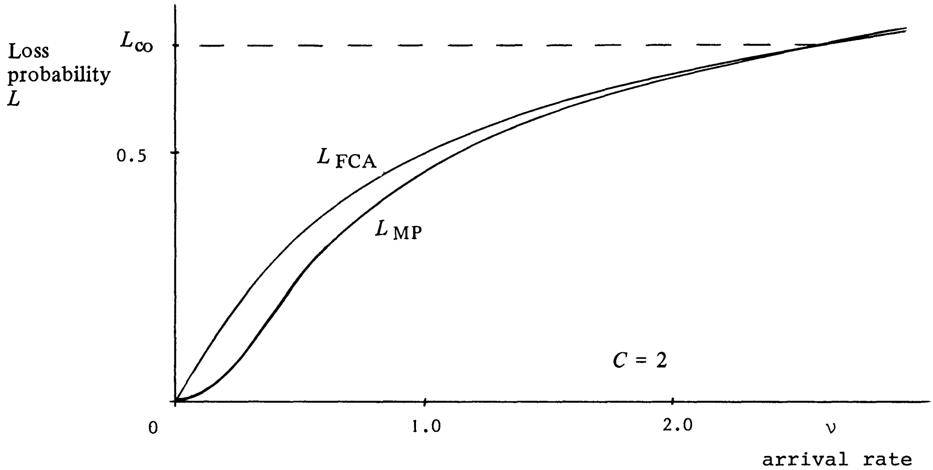


Fig. 4. Loss probabilities under fixed and dynamic channel assignment

observation from the case $C = 2$ is that as $\nu \to \infty$, $\pi(2) \to 0$ and $\pi(1) \to 1$. At high enough traffic intensities the allocation of channels to cells under the maximum packing strategy approaches that of a fixed channel assignment.

As $C$ increases the cross-over loss probability $L_{co}$ (illustrated in Fig. 4) decreases to zero. This is perhaps even more surprising, and so we sketch a proof. Hold the loss probability $L = L_{MP} = L_{FCA}$ fixed at an arbitrary level $L$, by choosing $\nu = \nu_{MP}$ or $\nu = \nu_{FCA}$ as a function of $C$. Then (cf. Kelly, 1986b) under the maximum packing strategy the distribution of $(C - n_v - n_{v+1}, v = -s, -s+1, \ldots, s)$ converges weakly, as $C \to \infty$, to the distribution of a vector $(m_{-s}, m_{-s+1}, \ldots, m_s)$ of independent random variables, $m_v$ geometrically distributed with parameter $(1-L)^{1/2}$. Moments converge also, and so

$$E(n_v + n_{v+1}) = C - (1-L)^{1/2}\,[1-(1-L)^{1/2}]^{-1} + o(1) \quad \text{as} \quad C \to \infty$$

The mean number of idle channels per cell is thus

$$\tfrac{1}{2}\,(1-L)^{1/2}\,[1-(1-L)^{1/2}]^{-1} + o(1) \quad \text{as} \quad C \to \infty. \tag{3.5}$$

Under a fixed channel assignment the mean number of idle channels per cell is

$$(1-L)\,L^{-1} + o(1) \quad \text{as} \quad C \to \infty. \tag{3.6}$$

For $C$ large enough, expression (3.6) is less than expression (3.5). It follows that if $C$ is large enough then $\nu_{MP} < \nu_{FCA}$. This holds for any $L > 0$ and so, as $C$ increases, the cross-over loss probability $L_{co}$ must decrease to zero. Note that the difference between expression (3.5) and (3.6) is of order $O(1)$ as $C \to \infty$, and thus any inferiority of the maximum packing strategy relative to a fixed channel assignment is very slight.

Methods of dynamic channel assignment should not be expected to perform better than a fixed channel assignment for fixed traffic intensities. The real advantage of dynamic schemes is their ability to cope automatically with traffic intensities which fluctuate in space and time. Kelly (1985b) describes a method of estimating loss probabilities under the maximum packing strategy for two-dimensional layouts with spatially varying traffic intensities. Many other schemes for dynamic channel assignment have been proposed: see, for example, the simulation study by Kahwa and Georganas (1978) of hybrid schemes, in which some channels are fixed and some dynamic.

## 4. CONTROL OF A RANDOM ACCESS BROADCAST CHANNEL

Consider a large number of stations able to communicate with each other over a single channel. When a station transmits a message it is heard by all stations, and in particular by the stations to whom it is addressed. However if two or more messages overlap then they interfere and must be retransmitted. How should retransmissions be scheduled? The first method used was the ALOHA scheme. This was developed for a radio broadcast network connecting terminals on the Hawaiian islands to a central computing facility, but it is simpler to explain how the scheme operates for a satellite channel (Kleinrock, 1976). The round-trip transmission time from a ground-station to a satellite transponder in geosynchronous orbit and back to earth is about a quarter of a second, much longer than a message length. The ALOHA scheme allows a station to transmit as soon as it has a message ready to send. If after one propagation delay the station hears its uncorrupted transmission it assumes no conflict has occurred. Otherwise it must retransmit. If all the stations involved in a conflict retransmit immediately they will conflict again, and so instead each station delays its retransmission for a random period. Each station repeats this procedure until its message is successfully transmitted.

Propagation delays are important even in local area networks. For Ethernet† (Metcalfe and Boggs, 1976; Digital, 1982b), where the medium is a coaxial cable less than 3 kilometres long, a station may have transmitted hundreds of bits before the first bit has traversed the network. Messages are usually longer than this and so in local area networks CSMA/CD (carrier sense multiple access with collision detect) schemes are used. When a station has a message to send it listens to the channel, and does not begin to transmit until the channel appears idle. Within a slot, whose length is the round-trip propagation delay, the station will know whether it has the channel to itself or whether it must back off and try again later.

It is important to know if a random access scheme can effectively control the use of the channel, or if instead the channel becomes clogged with collisions. To investigate this question we shall consider the following model. Assume there is an infinite number of stations and that new packets for transmission arrive in a Poisson stream of rate $\nu < 1$ from time $t = 0$ onwards. Assume that no station ever has more than one packet to transmit (we shall comment on this assumption later). Let the time axis be slotted so that one packet can be successfully transmitted in the slot $(t, t+1]$, $t = 1, 2, \ldots$. Let $Z_t$ represent the channel output during the slot $(t, t+1]$. Then $Z_t = 0, 1$ or $*$ depending on whether zero, one or more than one transmissions are attempted during slot $(t, t+1]$. We first model the ALOHA scheme. Let $Y_{t-1}$ be the number of packets that arrive during the slot $(t-1, t]$. These packets are first transmitted in the slot $(t, t+1]$, the first complete slot after their arrival. Also, packets which arrived earlier but have not yet been successfully transmitted are independently retransmitted with probability $f \in (0, 1)$ in the slot $(t, t+1]$. Thus the retransmission delay following an unsuccessful attempt is geometrically distributed with parameter $1-f$. Let the backlog $N_t$ be the number of stations with packets at time $t$. Then $N_t$ is a Markov chain, with

$$N_{t+1} = N_t + Y_t - I[Z_t = 1].$$

Here $I[A]$ is the indicator function of the event $A$. Thus

$$E(N_{t+1} - N_t \mid N_t = n) = \nu - P\{Z_t = 1 \mid N_t = n\}, \tag{4.1}$$

where

$$P\{Z_t = 1 \mid N_t = n\} = e^{-\nu} nf(1-f)^{n-1} + \nu e^{-\nu}(1-f)^n. \tag{4.2}$$

The drift (4.1) is thus positive for values of $n$ above a threshold. Typical simulation behaviour is that for a period, which can be extremely long, the system appears stable, but that eventually the backlog leaves a safe region and grows without bound (Kleinrock, 1976). Fayolle *et al.* (1977), and Rosenkrantz and Towsley (1983) have shown that the Markov chain $N_t$ is transient;

† Ethernet is a trademark of the Xerox Corporation

this also follows from the result we are about to obtain. Let

$$p(n) = P\{\exists\, T \geqslant 1: Z_T = 0 \text{ or } 1, N_1, N_2, \ldots, N_T \leqslant n \mid N_1 = n\}$$

be the probability that the channel unjams before the backlog increases from $n$. To calculate $p(n)$ suppose that $Z_1, Z_2, \ldots, Z_{t-1} = *, N_t = n$. Then *either* $Z_t = 0$ or 1, in which case set $T = t$; *or* $Z_t = *$, $N_{t+1} > n$, in which case no $T$ exists; *or* $Z_t = *$, $N_{t+1} = n$, in which case move to time $t+1$, with $Z_1, Z_2, \ldots, Z_t = *, N_{t+1} = n$. Thus

$$\begin{aligned} p(n) &= \frac{P\{Z_t = 0 \text{ or } 1 \mid N_t = n\}}{1 - P\{N_{t+1} = n, Z_t = * \mid N_t = n\}} \\ &= \frac{e^{-\nu}(1+\nu)(1-f)^n + e^{-\nu} nf(1-f)^{n-1}}{1 - e^{-\nu}[1-(1-f)^n - nf(1-f)^{n-1}]} \\ &\sim \frac{nf(1-f)^{n-1}}{e^{-\nu}-1} \quad \text{as} \quad n \to \infty. \end{aligned}$$

Hence $\Sigma p(n) < \infty$. Next consider the times at which $N_t$ reaches record values: set $R(1) = 1$ and

$$R(r+1) = \min\{t > R(r): N_t > N_{R(r)}\} \quad (r = 1, 2, ..).$$

With probability one this recursion defines an infinite sequence $R(r)$, $r = 1, 2, \ldots$; then $N_{R(r)}$, $r = 1, 2, \ldots$, is a strictly increasing sequence, and

$$\sum_{r=1}^{\infty} p(N_{R(r)}) < \infty.$$

Thus, by the Borel-Cantelli lemma,

$$P\{\exists\, J < \infty: Z_t = *, t \geqslant J\} = 1.$$

With probability one the channel transmits successfully only a finite number of messages and then jams itself forever, for any $\nu > 0$. Indeed it is possible to establish a stronger result: the time until the channel jams forever has finite expectation.

If it were possible to choose the retransmission probability $f = f_n$ as a function of the backlog $N_t = n$ then the choice maximizing expression (4.2) would be

$$f_n = \frac{1-\nu}{n-\nu}.$$

With this choice the drift (4.1) becomes

$$\begin{aligned} E(N_{t+1} - N_t \mid N_t = n, f = f_n) &= \nu - e^{-\nu}\left(\frac{n-1}{n-\nu}\right)^{n-1} \\ &\to \nu - e^{-1} \quad \text{as} \quad n \to \infty. \end{aligned}$$

The resulting Markov chain $N_t$ is positive recurrent for $\nu < e^{-1}$ (by Foster's criterion, Tweedie, 1976) and (since its increments have bounded second moment) transient for $\nu > e^{-1}$. But stations do not know the size of the backlog $N_t$, since their only means of communication is the channel itself. Mikhailov (1979) and Hajek and van Loon (1982) have devised schemes which allow stations to choose the transmission probability $f_t = f(Z_1, Z_2, \ldots, Z_{t-1})$ as a function of the past channel output. We now describe one such scheme. Suppose that every station maintains a counter $S_t$, updated by the recursion

$$S_{t+1} = \max\{1, S_t + aI[Z_t = 0] + bI[Z_t = 1] + cI[Z_t = *]\}.$$

For example, if $a = -1$, $b = 0$, $c = 1$ then $S_t$ decreases or increases by one according as the last slot was empty or the occasion of a clash. Suppose that each station with a packet to transmit does so with probability $f_t = S_t^{-1}$. Then $(N_t, S_t)$ is a Markov chain. We would like $S_t$ to track the backlog $N_t$, at least when $N_t$ is large. Consider the drift of $S_t$ when $N_t = n$ is large, with $\kappa = n/s$ held fixed.

$$E(S_{t+1} - S_t \mid N_t = n, S_t = s) = (a-c)\left(1 - \frac{1}{s}\right)^n + (b-c)\frac{n}{s}\left(1 - \frac{1}{s}\right)^{n-1} + c$$

$$\to (a-c)\, e^{-\kappa} + (b-c)\,\kappa e^{-\kappa} + c \quad \text{as} \quad n \to \infty. \tag{4.3}$$

The choice $a = (2-e)\,c$, $b = 0$, $c > 0$, makes the drift (4.3) negative if $\kappa < 1$ and positive if $\kappa > 1$. Thus if the backlog were held steady at a large value $N = n$, the counter $S$ would approach that value. But the backlog fluctuates, and it is important that $S_t$ can move quickly enough to keep up. Now

$$E(N_{t+1} - N_t \mid N_t = n, S_t = s) = \nu - \frac{n}{s}\left(1 - \frac{1}{s}\right)^{n-1}$$

$$\to \nu - \kappa e^{-\kappa} \quad \text{as} \quad n \to \infty. \tag{4.4}$$

When $\kappa > 1$ the drift components (4.3) and (4.4) will push $\kappa$ $(= N_t/S_t)$ towards unity provided we ensure that

$$\nu - \kappa e^{-\kappa} < \kappa[(a-c)\, e^{-\kappa} + (b-c)\,\kappa e^{-\kappa} + c]$$

For example, if $(a, b, c) = (2-e, 0, 1)$ then this condition holds for all $\kappa > 1$, provided $\nu < e^{-1}$. This heuristic discussion is intended merely to motivate a particular choice for the constants $(a, b, c)$. Using the elegant and powerful geometrical analysis of Mikhailov (1979) it is possible to establish that the choice $(a, b, c) = (2-e, 0, 1)$ produces a Markov chain $(N_t, S_t)$ which is positive recurrent whenever $\nu < e^{-1}$. Observe that schemes can be constructed to cope with certain limitations on the information available from the channel output: for example the choice $(a, b, c) = 1 - e/2, 1 - e/2, 1)$ results in a scheme which only requires stations to distinguish collisions from non-collisions. Hajek and van Loon (1982) have considered multiplicative recursions of the form

$$S_{t+1} = \max\{1, a(Z_t).\, S_t\}$$

for constants $a(0), a(1), a(*)$. Hajek (1982a) has shown that, provided $\nu < e^{-1}$, there exists a choice for these constants such that the resulting Markov chain $(N_t, S_t)$ is geometrically ergodic with a stationary distribution under which

$$E(N_t^k) < \infty \quad \text{for all} \quad k > 0.$$

The bound $e^{-1} \approx 0.368$ is a consequence of the assumption that all stations use the same transmission probability $f_t$. More complex schemes are possible, and Capetanakis (1979), Tsybakov and Mikhailov (1978) have proposed algorithms which can achieve stable throughputs higher than $e^{-1}$. An indication of their method follows. In the first slot all active stations transmit. If there is a conflict, those stations involved flip independent coins. Stations with tails hold off until stations with heads have all transmitted successfully; stations with heads transmit and repeat the procedure. Arriving packets wait until the initial conflict has been resolved, and are all transmitted in the first slot of the next cycle of the algorithm. An upper bound on the throughput of such conflict resolution algorithms is 0.587 (Tsybakov and Mikhailov, 1981) and algorithms are known that achieve a throughput of 0.488: see Berger (1981), Massey (1981) and Hajek

(1982b) for reviews, and Berger *et al.* (1984) for a discussion of the relationship between conflict resolution algorithms and the group testing problem (Hwang, 1976). Conflict resolution algorithms, however, seem likely to be more sensitive to any corruption of the feedback information $(Z_1, Z_2, \ldots, Z_{t-1})$ than schemes of the Mikhailov-Hajek-van Loon type.

If messages are longer than one slot length a station can use the first packet it successfully transmits to reserve slots for the rest of the message. If $M$ is the mean number of slots required by a message then those schemes we have discussed which can achieve a stable throughput of up to $\eta$ when $M = 1$ will, with a reservation procedure, be able to achieve a stable throughput of up to $M(M + \eta^{-1} - 1)^{-1}$. Thus the parameter $M$ will usually be more important than $\eta$, provided the scheme used is stable for *some* positive throughput.

The stable schemes we have discussed make essential use of the channel output $(Z_1, Z_2, \ldots, Z_{t-1})$. We shall now consider protocols in which a station does not use this information, but relies instead on the history of its own transmission attempts. Such protocols are called *acknowledgement based* (or collision detect) random access schemes. In Ethernet, for example, a station which has attempted unsuccessfully to transmit a packet $r$ times retransmits after a period with a discrete uniform distribution on $B_r = \{1, 2, 3, \ldots, 2^{\min\{10, r\}}\}$ until $r = 16$, and then discards the packet (the truncated binary exponential backoff algorithm; Digital, 1982a). Discarding packets ensures recurrence, but interesting theoretical questions concern the stability of the scheme when $B_r = \{1, 2, 3, \ldots, 2^r\}$ for all $r > 0$, with no discards. In particular, in view of the delays introduced, is it necessary that the backoff grow exponentially?

Suppose that a station at which a packet arrives in slot $(t, t + 1]$ attempts transmission in slots $(t + x_r, t + x_r + 1]$, $1 = x_1 < x_2 < x_3 \ldots$, until the packet is successfully transmitted, where $X = \{x_1, x_2, \ldots\}$ is a random set. Assume the choice of the set $X$ is independent from packet to packet and of the arrival process. Let $h(x) = P\{x \in X\}$. Thus $h(1) = 1$, $h(x) = f$, $x = 2, 3, \ldots$ for the ALOHA scheme. For the purpose of argument suppose now that the channel is externally jammed from time $t = 1$ onwards, so that no packets are successfully transmitted. The number of transmission attempts which are made in slot $(t, t + 1]$ has a Poisson distribution with mean $\nu\{h(1) + h(2) + \ldots + h(t)\}$. The probability of less than two attempts in slot $(t, t + 1]$ is thus

$$\left\{1 + \nu \sum_{x=1}^{t} h(x)\right\} \exp\left\{-\nu \sum_{x=1}^{t} h(x)\right\}.$$

Let $\Phi$ be the set consisting of those slots in which less than two attempts are made. The expected number of such slots, $E(|\Phi|)$, is

$$H(\nu) = \sum_{t=1}^{\infty} \left[\left\{1 + \nu \sum_{x=1}^{t} h(x)\right\} \exp\left\{-\nu \sum_{x=1}^{t} h(x)\right\}\right].$$

Let

$$\nu_c = \inf\{\nu : H(\nu) < \infty\}.$$

Thus if $\nu > \nu_c$ then $E(|\Phi|) < \infty$, and hence $\Phi$ is finite with probability one.

Remove now the supposition that the channel is externally jammed, and suppose that $\nu > \nu_c$. Subject the system to an additional, independent, Poisson arrival stream of rate $\epsilon > 0$. There is a positive probability that the additional arrivals jam every slot in the set $\Phi$, since $\Phi$ is finite with probability one. There is thus a positive probability that in *every* slot two or more transmissions are attempted. From this it follows (and this deduction is due to Iain MacPhee) that the number of successful transmissions is bounded above by a random variable with a geometric distribution, and hence the expected number of successful transmissions is finite. These conclusions hold for a system with Poisson arrivals at rate $\nu + \epsilon$. Since $\nu$ and $\epsilon$ are arbitrary subject to $\nu > \nu_c$, $\epsilon > 0$ it follows that for a system with arrival rate $\nu > \nu_c$ the expected number of successful transmissions is finite.

Suppose now that $\nu_c > 0$, and consider an arrival rate $\nu \in (0, \nu_c)$. Let $\Psi_i$ be the set consisting of those slots where $i$ retransmission attempts are made, for $i = 0$, 1. Then $\Phi \subset \Psi_0 \cup \Psi_1$ (the inclusion may be strict, since the set $\Phi$ was defined assuming the channel externally jammed, and since the definition of $\Psi_0$, $\Psi_1$ does not involve first transmission attempts). Since $\nu \in (0, \nu_c)$ we know that $E(|\Phi|) = \infty$, and hence $E(|\Psi_0|) + E(|\Psi_1|) = \infty$. Let $Y_t$ be the number of first transmission attempts in slot $(t, t+1]$. Then the expected number of successful transmissions is

$$E\left[\sum_{t=1}^{\infty} \{I[(t,t+1] \in \Psi_1, Y_t = 0] + I[(t,t+1] \in \Psi_0, Y_t = 1]\}\right]$$

$$= e^{-\nu} E\left[\sum_{t=1}^{\infty} I[(t,t+1] \in \Psi_1]\right] + \nu e^{-\nu} E\left[\sum_{t=1}^{\infty} I[(t,t+1] \in \Psi_0]\right]$$

$$= e^{-\nu} E(|\Psi_1|) + \nu e^{-\nu} E(|\Psi_0|) = \infty.$$

Hence if $\nu \in (0, \nu_c)$ the expected number of successful transmissions is infinite.

For the ALOHA scheme, $\nu_c = 0$. More generally, $\nu_c = 0$ whenever

$$(\log t)^{-1} \sum_{x=1}^{t} h(x) \to \infty \quad \text{as} \quad t \to \infty. \tag{4.5}$$

For the Ethernet scheme with $B_r = \{1, 2, 3, \ldots, \lfloor b^r \rfloor\}$ and no discards

$$\sum_{x=1}^{t} h(x) \sim \log_b t \quad \text{as} \quad t \to \infty,$$

and hence $\nu_c = \log b$. For example, if $b = 2$ then $\nu_c = 0.693...$ This result does *not* imply that for $\nu < \nu_c$ the Ethernet scheme is strongly stable, in the sense that there exists a positive recurrent Markov process describing the evolution of the system. To emphasize this point, suppose there does exist such a process, and let $\pi_i$ be its equilibrium probability of retransmitting $i$ packets in slot $(t, t+1]$, for $i = 0, 1$. Then, equating arrival and departure rates,

$$\nu = \pi_1 e^{-\nu} + \pi_0 \nu e^{-\nu}.$$

Since $\pi_0 + \pi_1 < 1$ it follows that $\nu < e^{-\nu}$, and hence that $\nu < 0.567..$ (this argument applies more widely than acknowledgement based schemes—it requires only that a packet is first transmitted in the slot following its arrival). Thus for $b = 2$ and $0.567.. < \nu < 0.693..$ the Ethernet scheme is not strongly stable, yet the expected number of successful transmissions is infinite. It is an open question whether there exist any strongly stable acknowledgement based random access schemes (for other work on this problem see Hajek, 1982c, and Rosenkrantz, 1984). Condition (4.5) does however show that the expected number of successful transmissions is finite for any acknowledgement based scheme with slower than exponential backoff.

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