

What are imsets and what they are good for

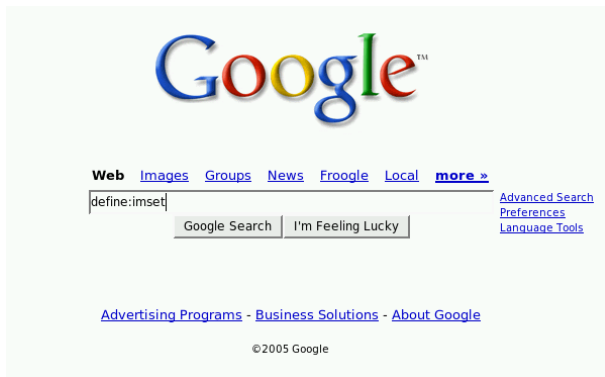
Jiří Vomlel and Milan Studený

ÚTIA AV ČR

VŘSR, 11. - 13. 11. 2005

What is an imset?

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In Egyptian mythology **Imset** was a funerary deity, one of the Four sons of Horus, who were associated with the canopic jars, specifically the one which contained the liver.



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What is an imset? (formal definition)

- N ... a finite set
- $\mathcal{P}(N)$... power set of N
- \mathbb{Z} ... set of all integers

Definition

Imset u is a function $u : \mathcal{P}(N) \mapsto \mathbb{Z}$.

Function $m : \mathcal{P}(N) \mapsto \mathbb{N}$ is sometimes called multiset. Thus, **imset** is an abbreviation from **I**nteger valued **M**ulti**S**ET. Studený (2001)

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What is an imset? (an example)

Let $N = \{a, b, c\}$. An imset u over N is

\emptyset	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{b, c\}$	$\{a, c\}$	$\{a, b, c\}$
0	0	+1	0	-1	-1	0	+1

A convention:

$$\delta_A(B) = \begin{cases} 1 & \text{if } A = B \\ 0 & \text{otherwise} \end{cases}$$

$$\forall B \subseteq N: u(B) = \sum_{A \subseteq N} u(A) \cdot \delta_A(B)$$

$$u = \sum_{A \subseteq N} c_A \cdot \delta_A$$

Using the convention we will write

$$u = \delta_{\{b\}} - \delta_{\{a,b\}} - \delta_{\{b,c\}} + \delta_{\{a,b,c\}}$$

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Elementary and structural imsets

Definition (Elementary imset)

Let $K \subseteq N$, $a, b \in N \setminus K$, and $a \neq b$.

Elementary imset is defined by the formula

$$u_{\langle a,b|K \rangle} = \delta_{\{a,b\} \cup K} + \delta_K - \delta_{\{a\} \cup K} - \delta_{\{b\} \cup K}$$

Let $\mathcal{E}(N)$ denote the set of all elementary imsets.

Definition (Structural imset)

An imset u is **structural** iff

$$n \cdot u = \sum_{v \in \mathcal{E}(N)} k_v \cdot v,$$

where $n \in \mathbb{N}$ and $v \in \mathcal{E}(N) : k_v \in \mathbb{N} \cup \{0\}$

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Conditional Independence (CI) Models (definition)

Definition (Disjoint triplet over N)

Let $A, B, C \subseteq N$ be pairwise disjoint. Then $\langle A, B \mid C \rangle$ denotes a **disjoint triplet over N** . $\mathcal{T}(N)$ will denote the class of all possible disjoint triplets over N .

Definition (CI-statement)

Let $\langle A, B \mid C \rangle$ be a disjoint triplet over N . Then “ A is conditionally independent of B given C ” is an **CI-statement**, written as $A \perp\!\!\!\perp B \mid C$.

Definition (CI-model)

CI-model is a set of CI-statements.

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CI-model is a set of CI-statements.

CI-model (an example by F. V. Jensen)

Assume three variables:

- h is the length of hair *long, short*,
- s is stature that has states $< 168\text{cm}, > 168\text{cm}$,
- g is gender that takes states *male, female*.

If we do not know the gender of a person

- seeing the length of his/her hair will tell us more about the gender, i.e. $\text{not}(h \perp\!\!\!\perp g)$,
- this will focus our belief on his/her stature, i.e. $\text{not}(g \perp\!\!\!\perp s)$ and $\text{not}(h \perp\!\!\!\perp s)$.

But if we **know the gender** of a person then **length of hair** gives us no extra clue on his **stature**, i.e., $h \perp\!\!\!\perp s \mid g$.

Thus, we get a CI-model consisting of just one CI-statement $h \perp\!\!\!\perp s \mid g$.

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CI-objects

Definition (CI-object)

CI-object over N is a mathematical object defined over N that can be used to generate a CI-model.

Classes of CI-objects

- 1 Probability distributions (PDs) from a given class defined over random variables from N . E.g., discrete PDs over N .
- 2 Undirected graphs (UGs) over the set of nodes N .
- 3 Acyclic directed graph (DAGs) over the set of nodes N .
- 4 Structural imsets over N .

CI-objects generating the CI-model $a \perp\!\!\!\perp c \mid b$

- 1 $\forall v_a, v_b, v_c:$
$$P(a = v_a, c = v_c \mid b = v_b) = P(a = v_a \mid b = v_b) \cdot P(c = v_c \mid b = v_b)$$
- 2 $a - b - c$
- 3 $a \leftarrow b \rightarrow c$ but also $a \leftarrow b \leftarrow c$ and $a \rightarrow b \rightarrow c$
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How CI-objects generate CI-models?

In UGs

Definition (Separation criteria)

$A \perp\!\!\!\perp B \mid C$ is represented in UG G if every route in G between a node in A and a node in B contains a node from C .

In imsets

Definition

$A \perp\!\!\!\perp B \mid C$ is represented in an imset u if there exists $k \in \mathbb{N}$ such that

$$k \cdot u = u_{\langle A, B \mid C \rangle} + w$$

where w is a structural imset.

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Perfectly Markovian probability distribution

- $M(P)$... a CI-model generated by a **discrete probability distribution** P over N
- \mathcal{M} ... the class of all CI-models generated by a **discrete probability distribution** over N
- \mathcal{O} ... a considered class of CI-objects (e.g., DAGs, UGs, structural imsets)

Definition (Perfectly Markovian)

A probability distribution P is perfectly Markovian with respect to an object $O \in \mathcal{O}$ if for every $\langle A, B \mid C \rangle \in \mathcal{I}(N)$

$A \perp\!\!\!\perp B \mid C$ is represented in $P \iff A \perp\!\!\!\perp B \mid C$ is represented in M .

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- $M(P)$... a CI-model generated by a **discrete probability distribution** P over N
- \mathcal{M} ... the class of all CI-models generated by a **discrete probability distribution** over N
- \mathcal{O} ... a considered class of CI-objects (e.g., DAGs, UGs, structural imsets)

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A probability distribution P is perfectly Markovian with respect to an object $O \in \mathcal{O}$ if for every $\langle A, B \mid C \rangle \in \mathcal{I}(N)$

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Properties of classes of CI-objects (definitions)

Definition (Faithfulness of \mathcal{O})

For every CI-object $O \in \mathcal{O}$ there exists a CI-model $M(P)$ from \mathcal{M} such that P is perfectly Markovian with respect to O .

Definition (Completeness of \mathcal{O})

For every CI-model $M(P)$ from \mathcal{M} there exists a CI-object $O \in \mathcal{O}$ such that P is perfectly Markovian with respect to O .

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For every CI-model $M(P)$ from \mathcal{M} there exists **at most one** CI-object $O \in \mathcal{O}$ such that P is perfectly Markovian with respect to O .

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4	64	185	18300

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