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CALCULATING THE MINIMAX ESTIMATOR OF A BINOMIAL PROBABILITY WITH ENTROPY LOSS FUNCTION AND ITS COMPARISON WITH OTHER ESTIMATORS OF A BINOMIAL PROBABILITY

R. Wieczorkowski

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Abstract

This paper presents the numerical algorithm for calculating the minimax estimator of a binomial probability with entropy loss function. We use this algorithm to tabulate minimax estimator for n from 2 to 50. In the second part of the paper, this minimax estimator is statistically compared with other known estimators of a binomial probability by Monte Carlo simulation.

1. Introduction

Let X be a random variable such that

$$P_\theta\{X = x\} = \binom{n}{x} \theta^x (1-\theta)^{n-x}, x = 0, 1, ..., n$$

where $\theta \in [0, 1]$ is the unknown parameter. The problem is to estimate θ under entropy loss function given by the formula

$$L(\theta, d) = \sum_{x=0}^{n} \left(\log \frac{P_\theta\{X = x\}}{P_d\{X = x\}} \right) P_\theta\{X = x\} = n[\theta \log \frac{\theta}{d} + (1-\theta) \log \frac{(1-\theta)}{(1-d)}]$$

where $d \in [0, 1]$ is estimator of θ ($\gamma \log 0$ is defined to be 0 and ∞ for $\gamma = 0$ and $\gamma < 0$, respectively). The entropy loss function is a more convincing loss function for estimation of the binomial parameter (for detailed discussion see e.g. [3] and [9]).

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Let $\delta : \{0, 1, ..., n\} \to [0, 1]$ be an estimator and let D be the class of all estimators. The risk function of the estimator δ is denoted by $R(\theta, \delta)$. The risk function of the estimator $\delta = (\delta_0, \delta_1, ..., \delta_n)$, where $\delta_i = \delta(i)$, is given by the formula

$$R(\theta, \delta) = n \sum_{x=0}^{n} \binom{n}{x} \left[\theta \log \frac{\theta}{\delta_x} + (1-\theta) \log \frac{1-\theta}{1-\delta_x}\right] \theta^x (1-\theta)^{n-x}.$$

Denote $\rho(\delta) = \sup_{0 \le \theta \le 1} R(\theta, \delta)$. The minimax estimator δ minimizes $\rho(\delta)$.

Let $D_m = \{\delta \in D : \delta_x \le \delta_y$ whenever $x \le y\}$ be the class of all monotone estimators and let $D_s = \{\delta \in D : \delta_x = 1 - \delta_{n-x}, x = 0, 1, ..., n\}$ be the class of all symmetric estimators. According to results proved in [8] the minimax estimator under entropy loss function is the estimator minimizing $\rho(\delta)$ over $\delta \in D_m \cap D_s$. This estimator is also admissible. These results reduce our problem to the problem of constrained optimization, which can be solved only numerically.

2. Algorithm for calculating minimax estimator

For the numerical calculation of the minimax estimator, we have used the "complex method" introduced by Box in [1]. This is an iterative algorithm for constrained minimization, which is effective for convex functions defined over convex domains (which is our case, moreover the function $\rho(\delta)$ is strictly convex and the estimator is unique). The implementation of Box algorithm in Basic may be found in [2]. Our optimization problem can be numerically solved by many known constrained minimization algorithms. We have decided to use the Box method because of its accessibility and ease of implementation.

The complex method is a modification of "downhill simplex method" of Nelder and Mead [5] taking into account constraints over variables. The simplex method is an algorithm for multidimensional minimization (in R^k) which requires only function (not its derivatives) evaluations. This algorithm must be started with $k+1$ points, vertices of geometrical figure called simplex. The main loop of the algorithm consists of a sequence of transformations. Possible transformations of the simplex are: a reflection away from the high point, a reflection and expansion away from the high point, a contraction along one dimension from the high point, a contraction along all dimensions towards the low point. For an appropriate sequence of such steps, the simplex will converge to a minimum of the function. For detailed discussion of this algorithm see [6] . The simplex method is not very efficient in terms of the number of function evaluations but has a natural geometrical interpretation and is easy to implement.

In the complex method due to Box, the simplex is replaced by the so called complex, which is a set of $2k$ points in the feasible domain of a function. In the beginning of the algorithm one starting point must be given, and the other $2k-1$ points of the complex are formed using constraints on variables and a pseudorandom number generator.

Our minimization problem: to minimize $\rho(\delta)$ over $\delta \in D_m \cap D_s$, can be equivalently formulated in the form used in the complex algorithm

$\min f(x_0, x_1, ..., x_{k-1}),$

where $k = \lfloor n/2 \rfloor$, $0 \le x_i \le 0.5, i = 0, 1, ..., k-1$

and $x_{i-1} \le x_i, i = 1, 2, ..., k-1$, with
$f(x_0, x_1, ..., x_{k-1}) = \rho(x_0, x_1, ..., x_{k-1}, 1-x_{k-1}, 1-x_{k-2}, ..., 1-x_1, 1-x_0)$.
As the initial point we have chosen $x = (x_0, x_1, ..., x_{k-1})$, $x_i = \frac{i+1}{2k}$, $i = 0, 1, ..., k-1$.
To calculate the values of the function $\rho(\delta)$ for given δ we have used the following procedure:

- The zeros of the function $\frac{dR(\theta,\delta)}{d\theta}$ in the interval $[0,1]$ are found;
- These zeros are used to obtain $\rho(\delta) = \sup_{0 \le \theta \le 1} R(\theta, \delta)$.

This procedure uses the fact that the number of points where the first derivative of $R(\theta, \delta)$ vanishes (i.e. where the local extrema are attained) is bounded from above by number of zeros of the second derivative, which yields a number less then $n+3$ (see [7]). Calculating all roots of the equation $\frac{dR(\theta,\delta)}{d\theta} = 0$ was carried out by a simple procedure of step by step approximation. The roots were calculated with absolute error less than 10^{-6}.

3. Tables of ELF estimator

Tables 1-5 present results of numerical evaluation of the minimax estimator under the entropy loss function (ELF). The absolute error of the results in the tables is less than 10^{-3}. The algorithm was written in C language and calculations were performed on PC486 under *Linux* operating system.

4. Statistical comparison

This part of the paper deals with the comparison of the performance of the calculated minimax estimator with other known estimators of binomial probability. Estimators are compared in terms of the root mean squared error (RMSE) and Pitman's measure of closeness (PC) evaluated by Monte Carlo simulation.

4.1. Other estimators

In our experiment we have used the following estimators of a binomial probability:

- ELF estimator, determined in the first part of the paper;
- MLE estimator, this estimator has form $\delta_i = \frac{i}{n}$ and is obtained by the maximum likelihood estimation method;
- SLF estimator: the minimax estimator under the squared loss function, $\delta_i = \frac{i+\frac{\sqrt{n}}{2}}{n+\sqrt{n}}$, which is obtained as the Bayesian estimator with beta prior;
- ASM estimator: the asymptotic linear minimax estimator under entropy loss function (see [7]), $\delta_i = \frac{i+\alpha}{n+2\alpha}$, where $\alpha = 0.509$.

4.2. Design of experiment

In our experiment the variability of estimator $\widehat{p}$ of the binomial probability p was measured by the root mean squared error (RMSE). The RMSE can be expressed as

$$RMSE = \sqrt{SE^2 + BIAS^2}, \tag{1}$$

where $BIAS = E(\widehat{p}) - p$ denotes the bias and $SE = \sqrt{Var(\widehat{p})}$ is the standard error of estimator. For estimators MLE, SLF and ASM it is easy to obtain explicit expressions for RMSE. To approximate RMSE for the ELF, Monte Carlo sampling experiments were performed. Eleven population cases for different values of parameter p ranging from 0.001 to 0.999 were considered. For each population case, $N = 100000$ random samples of size $5, 10, 15, 20, 30, 40$ and 50 from binomial distribution were generated, and then parameters p were estimated. The values of RMSE were then approximately computed using formula (1) , where $E(\widehat{p})$ and $Var(\widehat{p})$ were estimated by

$E(\widehat{p}) = \frac{1}{n}\sum_{i=1}^{N}\widehat{p}, \sqrt{Var(\widehat{p})} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(\widehat{p} - E(\widehat{p}))^2}.$

Here the summations are over N estimates of $\widehat{p}$ of p, with N being the number of random samples used in estimation ($N = 100000$). The precision of Monte Carlo approximations in most cases is greater than or equal 2 significant digits, which was confirmed by repeating simulations for estimators MLE, SLF and ASM with known analytical formulas for RMSE. In Monte Carlo simulations the generator from [4] was used.

As another method for comparing estimators we have used Pitman's measure of closeness (PC). For two estimators T_1 and T_2 this measure is defined by

$$PC(T_1, T_2) = P(|T_1 - p| < |T_2 - p|). \tag{2}$$

Pitman's measure of closeness for two competing estimators compares how frequently the loss due to one is less than the loss due to the other. We refer to a special issue of *Communication in Statistics, Part A – Theory and Methods (1991, Vol. 20, No. 11)* for detailed discussion of this notion. In our Monte Carlo experiment values of (2) were approximately computed using 100000 random samples of size $10, 20, 50$ from binomial distribution with different parameters p.

4.3. Result and discussion

The results of calculations of RMSE are shown in table 6. The table demonstrates that no estimator is superior according to RMSE. In general, it appears that no estimator is uniformly better than others. We observe that for a given parameter p (in most cases) the best estimator is the same for different values of n. What changes with p is only the kind of estimator. It is interesting that in all cases the estimator ELF is never the best, the estimator MLE is the best for values of p close to 0 or 1, and the estimator SLF performs best for central values of p . As the next best estimator we obtain the ELF estimator (for p in the central part of the interval $(0, 1)$), and the ASM estimator (for p close to 0 or 1).

Table 7. compares the
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Robert Wieczorkowski
Mathematical Institute
Warsaw University of Te
Zaklad Probabilistyki
Pl. Politechniki 1
00-661 Warszawa
e-mail: robw@alpha.im.p

Table 7. compares the ELF estimator with the other estimators by means of Pitman's measure of closeness. We observe again that there is no estimator which may be preferred uniformly for all $p \in (0,1)$. For values of p close to 0 or 1 the ELF estimator is evidently better compared with SLF and ASM, but worse compared with the MLE estimator. For values of parameter p in the central part of the interval $(0,1)$ (for example $p \in [0.3, 0.7]$) the values of Pitman's measure are concentrated near 0.5; in this situation none of considered estimators may be convincingly preferred.

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Robert Wieczorkowski
Mathematical Institute
Warsaw University of Technology
Zaklad Probabilistyki
Pl. Politechniki 1
00-661 Warszawa
e-mail: robw@alpha.im.pw.edu.pl

n	2	3	4	5	6	7	8	9	10
δ_0	.1523	.1114	.0902	.0764	.0656	.0577	.0516	.0466	.0425
δ_1	.5000	.4664	.3579	.3059	.2755	.2393	.2138	.1966	.1792
δ_2			.5000	.4042	.3421	.3112	.2777	.2463	.2277
δ_3					.5000	.4334	.3937	.3568	.3221
δ_4							.5000	.4629	.4214
δ_5									.5000

Table 1: ELF estimator, n from 2 to 10

n	11	12	13	14	15	16	17	18	19	20
δ_0	.0391	.0363	.0338	.0322	.0297	.0281	.0266	.0252	.0240	.0228
δ_1	.1647	.1497	.1429	.1317	.1255	.1172	.1104	.1094	.1042	.0960
δ_2	.2110	.1999	.1825	.1854	.1604	.1576	.1461	.1298	.1241	.1278
δ_3	.2961	.2763	.2547	.2196	.2285	.2011	.1986	.1962	.1875	.1654
δ_4	.3852	.3446	.3330	.3307	.2828	.2902	.2651	.2488	.2356	.2322
δ_5	.4619	.4407	.4024	.3906	.3635	.3196	.3051	.2875	.2726	.2657
δ_6		.5000	.4530	.4010	.4064	.3921	.3773	.3527	.3435	.3141
δ_7				.5000	.4541	.4367	.4051	.3960	.3674	.3620
δ_8						.5000	.4877	.4481	.4307	.4045
δ_9								.5000	.4777	.4575
δ_{10}										.4984

Table 2: ELF estimator, n from 11 to 20

n	21	22	23	24	25	26	27	28	29	30
δ_0	.0218	.0212	.0203	.0193	.0186	.0181	.0173	.0168	.0162	.0157
δ_1	.0930	.0828	.0782	.0847	.0795	.0745	.0695	.0703	.0694	.0674
δ_2	.1149	.1188	.1177	.1014	.0994	.1024	.0992	.0920	.0900	.0829
δ_3	.1671	.1620	.1527	.1492	.1412	.1306	.1265	.1304	.1207	.1219
δ_4	.2190	.2001	.1889	.1869	.1864	.1805	.1692	.1597	.1530	.1518
δ_5	.2498	.2372	.2334	.2287	.2096	.2099	.2076	.1917	.1972	.1821
δ_6	.3029	.3031	.2921	.2714	.2687	.2463	.2346	.2480	.2283	.2194
δ_7	.3448	.3244	.3092	.2935	.2831	.2900	.2721	.2633	.2467	.2510
δ_8	.3935	.3821	.3644	.3494	.3308	.3237	.3128	.2814	.2831	.2714
δ_9	.4261	.4058	.3886	.3912	.3740	.3432	.3413	.3377	.3273	.3116
δ_{10}	.4860	.4544	.4366	.4140	.4042	.3899	.3706	.3792	.3554	.3454
δ_{11}		.5000	.5000	.4605	.4447	.4423	.4267	.3847	.3882	.3788
δ_{12}				.4971	.4743	.4728	.4451	.4411	.4065	.4032
δ_{13}						.4996	.4683	.4586	.4593	.4306
δ_{14}								.4952	.4855	.4735
δ_{15}										.4944

Table 3: ELF estimator, n from 21 to 30

n	31	32	
δ_0	0.0152	0.0146	0.01
δ_1	0.0648	0.0627	0.06
δ_2	0.0841	0.0799	0.08
δ_3	0.1135	0.1105	0.10
δ_4	0.1444	0.1450	0.14
δ_5	0.1940	0.1751	0.17
δ_6	0.1952	0.2021	0.19
δ_7	0.2425	0.2317	0.21
δ_8	0.2682	0.2633	0.25
δ_9	0.3068	0.2906	0.28
δ_{10}	0.3294	0.3226	0.31
δ_{11}	0.3583	0.3518	0.32
δ_{12}	0.3978	0.3841	0.38
δ_{13}	0.4153	0.4082	0.41
δ_{14}	0.4588	0.4442	0.41
δ_{15}	0.4927	0.4644	0.44
δ_{16}			0.49
δ_{17}			
δ_{18}			
δ_{19}			
δ_{20}			

Table

n	31	32	33	34	35	36	37	38	39	40
δ_0	0.0152	0.0146	0.0144	0.0139	0.0134	0.0131	0.0129	0.0126	0.0123	0.0118
δ_1	0.0648	0.0627	0.0603	0.0576	0.0584	0.0547	0.0518	0.0566	0.0517	0.0495
δ_2	0.0841	0.0799	0.0814	0.0781	0.0739	0.0738	0.0744	0.0664	0.0726	0.0654
δ_3	0.1135	0.1105	0.1040	0.1027	0.0991	0.0974	0.0922	0.0934	0.0872	0.0907
δ_4	0.1444	0.1450	0.1423	0.1392	0.1348	0.1290	0.1303	0.1255	0.1162	0.1139
δ_5	0.1940	0.1751	0.1746	0.1607	0.1624	0.1559	0.1492	0.1485	0.1483	0.1455
δ_6	0.1952	0.2021	0.1996	0.1929	0.1836	0.1787	0.1726	0.1760	0.1745	0.1569
δ_7	0.2425	0.2317	0.2178	0.2264	0.2118	0.2133	0.2160	0.1922	0.1924	0.1938
δ_8	0.2682	0.2633	0.2569	0.2415	0.2395	0.2289	0.2160	0.2286	0.2056	0.2091
δ_9	0.3068	0.2906	0.2894	0.2693	0.2715	0.2614	0.2622	0.2409	0.2480	0.2365
δ_{10}	0.3294	0.3226	0.3167	0.3166	0.3020	0.2878	0.2704	0.2812	0.2759	0.2624
δ_{11}	0.3583	0.3518	0.3299	0.3287	0.3133	0.3174	0.3068	0.2990	0.2793	0.2805
δ_{12}	0.3978	0.3841	0.3808	0.3641	0.3538	0.3381	0.3334	0.3304	0.3312	0.3141
δ_{13}	0.4153	0.4082	0.4138	0.3762	0.3740	0.3702	0.3622	0.3379	0.3344	0.3277
δ_{14}	0.4588	0.4442	0.4168	0.4240	0.4094	0.3886	0.3797	0.3830	0.3693	0.3550
δ_{15}	0.4927	0.4644	0.4445	0.4440	0.4311	0.4219	0.4037	0.4017	0.3781	0.3869
δ_{16}			0.4916	0.4714	0.4552	0.4466	0.4377	0.4169	0.4093	0.4036
δ_{17}				0.4972	0.4936	0.4773	0.4667	0.4585	0.4464	0.4253
δ_{18}						0.4999	0.4810	0.4714	0.4739	0.4526
δ_{19}								0.4906	0.5000	0.4740
δ_{20}										0.4940

Table 4: ELF estimator, n from 31 to 40

n	41	42	43	44	45	46	47	48	49	50
δ_0	0.0116	0.0113	0.0111	0.0108	0.0107	0.0104	0.0102	0.0099	0.0098	0.0095
δ_1	0.0460	0.0463	0.0487	0.0464	0.0420	0.0434	0.0430	0.0421	0.0403	0.0417
δ_2	0.0673	0.0631	0.0578	0.0589	0.0637	0.0597	0.0555	0.0540	0.0558	0.0508
δ_3	0.0861	0.0855	0.0873	0.0838	0.0746	0.0756	0.0784	0.0763	0.0739	0.0743
δ_4	0.1102	0.1117	0.1087	0.1023	0.1063	0.1015	0.0969	0.0981	0.0936	0.0923
δ_5	0.1411	0.1302	0.1273	0.1322	0.1275	0.1259	0.1237	0.1168	0.1163	0.1146
δ_6	0.1601	0.1607	0.1600	0.1497	0.1448	0.1452	0.1419	0.1385	0.1434	0.1323
δ_7	0.1795	0.1803	0.1705	0.1722	0.1604	0.1617	0.1563	0.1595	0.1435	0.1517
δ_8	0.2120	0.2028	0.2049	0.1887	0.1980	0.1825	0.1833	0.1785	0.1856	0.1698
δ_9	0.2256	0.2197	0.2092	0.2209	0.2115	0.2082	0.2037	0.1929	0.1858	0.1906
δ_{10}	0.2559	0.2516	0.2492	0.2347	0.2241	0.2337	0.2286	0.2217	0.2137	0.2151
δ_{11}	0.2764	0.2706	0.2686	0.2577	0.2600	0.2473	0.2362	0.2420	0.2315	0.2225
δ_{12}	0.3056	0.2961	0.2831	0.2898	0.2762	0.2680	0.2679	0.2618	0.2575	0.2531
δ_{13}	0.3221	0.3209	0.3138	0.2986	0.2887	0.2856	0.2781	0.2734	0.2772	0.2684
δ_{14}	0.3423	0.3334	0.3287	0.3211	0.3222	0.3153	0.3096	0.2949	0.2883	0.2850
δ_{15}	0.3743	0.3688	0.3604	0.3514	0.3432	0.3377	0.3277	0.3240	0.3112	0.3085
δ_{16}	0.4033	0.3806	0.3648	0.3632	0.3596	0.3523	0.3476	0.3412	0.3325	0.3235
δ_{17}	0.4095	0.4114	0.4105	0.4001	0.3860	0.3745	0.3696	0.3617	0.3498	0.3503
δ_{18}	0.4375	0.4391	0.4180	0.4100	0.3965	0.3961	0.3838	0.3807	0.3734	0.3655
δ_{19}	0.4700	0.4445	0.4425	0.4308	0.4159	0.4128	0.3981	0.3930	0.3887	0.3805
δ_{20}	0.4953	0.4780	0.4675	0.4585	0.4589	0.4379	0.4298	0.4200	0.4164	0.4033
δ_{21}		0.4949	0.4948	0.4753	0.4734	0.4604	0.4573	0.4426	0.4281	0.4229
δ_{22}				0.4975	0.4828	0.4803	0.4687	0.4637	0.4487	0.4453
δ_{23}						0.4936	0.4931	0.4760	0.4743	0.4567
δ_{24}								0.4956	0.4930	0.4870
δ_{25}										0.4979

Table 5: ELF estimator, n from 41 to 50

n	p	MLE	SLF
5	0.001	0.014	0.155
	0.010	0.044	0.154
	0.100	0.134	0.155
	0.200	0.179	0.155
	0.300	0.205	0.155
	0.400	0.219	0.154
	0.500	0.224	0.154
	0.600	0.218	0.155
	0.700	0.205	0.154
	0.800	0.179	0.155
	0.900	0.134	0.155
	0.990	0.045	0.154
	0.999	0.014	0.155
10	0.001	0.010	0.120
	0.010	0.031	0.120
	0.100	0.095	0.120
	0.200	0.126	0.120
	0.300	0.145	0.120
	0.400	0.155	0.120
	0.500	0.158	0.120
	0.600	0.155	0.120
	0.700	0.145	0.120
	0.800	0.126	0.121
	0.900	0.095	0.120
	0.990	0.031	0.120
	0.999	0.010	0.120
20	0.001	0.007	0.091
	0.010	0.022	0.091
	0.100	0.067	0.091
	0.200	0.089	0.091
	0.300	0.102	0.091
	0.400	0.109	0.091
	0.500	0.112	0.092
	0.600	0.110	0.092
	0.700	0.102	0.091
	0.800	0.089	0.092
	0.900	0.067	0.091
	0.990	0.022	0.091
	0.999	0.007	0.091

Table 6: RM

n	p	MLE	SLF	ELF	ASM	n	p	MLE	SLF	ELF	ASM
5	0.001	0.014	0.155	0.078	0.085	30	0.001	0.006	0.077	0.018	0.017
	0.010	0.044	0.154	0.093	0.091		0.010	0.018	0.077	0.031	0.024
	0.100	0.134	0.155	0.152	0.130		0.100	0.055	0.077	0.056	0.055
	0.200	0.179	0.155	0.167	0.156		0.200	0.073	0.077	0.071	0.071
	0.300	0.205	0.155	0.169	0.174		0.300	0.084	0.077	0.080	0.082
	0.400	0.219	0.154	0.169	0.183		0.400	0.089	0.077	0.085	0.087
	0.500	0.224	0.154	0.169	0.186		0.500	0.091	0.077	0.086	0.088
	0.600	0.218	0.155	0.170	0.184		0.600	0.089	0.077	0.085	0.087
	0.700	0.205	0.154	0.169	0.174		0.700	0.084	0.077	0.080	0.081
	0.800	0.179	0.155	0.168	0.158		0.800	0.073	0.077	0.072	0.071
	0.900	0.134	0.155	0.152	0.130		0.900	0.055	0.077	0.056	0.055
	0.990	0.045	0.154	0.092	0.091		0.990	0.018	0.077	0.031	0.024
	0.999	0.014	0.155	0.078	0.085		0.999	0.006	0.077	0.018	0.017
10	0.001	0.010	0.120	0.045	0.047	40	0.001	0.005	0.068	0.014	0.013
	0.010	0.031	0.120	0.062	0.054		0.010	0.016	0.068	0.025	0.020
	0.100	0.095	0.120	0.105	0.094		0.100	0.047	0.068	0.048	0.047
	0.200	0.126	0.120	0.117	0.118		0.200	0.063	0.068	0.062	0.062
	0.300	0.145	0.120	0.128	0.133		0.300	0.073	0.068	0.070	0.071
	0.400	0.155	0.120	0.135	0.141		0.400	0.077	0.068	0.074	0.076
	0.500	0.158	0.120	0.137	0.143		0.500	0.079	0.068	0.076	0.077
	0.600	0.155	0.120	0.135	0.141		0.600	0.077	0.068	0.075	0.076
	0.700	0.145	0.120	0.129	0.132		0.700	0.072	0.069	0.070	0.071
	0.800	0.126	0.121	0.117	0.118		0.800	0.063	0.068	0.062	0.062
	0.900	0.095	0.120	0.105	0.094		0.900	0.048	0.068	0.049	0.047
	0.990	0.031	0.120	0.062	0.053		0.990	0.016	0.068	0.025	0.020
	0.999	0.010	0.120	0.045	0.047		0.999	0.005	0.068	0.014	0.013
20	0.001	0.007	0.091	0.025	0.025	50	0.001	0.004	0.062	0.012	0.011
	0.010	0.022	0.091	0.040	0.032		0.010	0.014	0.062	0.022	0.017
	0.100	0.067	0.091	0.069	0.067		0.100	0.042	0.062	0.043	0.042
	0.200	0.089	0.091	0.086	0.086		0.200	0.057	0.062	0.056	0.056
	0.300	0.102	0.091	0.096	0.098		0.300	0.065	0.062	0.063	0.063
	0.400	0.109	0.091	0.102	0.104		0.400	0.069	0.062	0.067	0.068
	0.500	0.112	0.092	0.104	0.106		0.500	0.071	0.062	0.068	0.069
	0.600	0.110	0.092	0.102	0.105		0.600	0.069	0.062	0.067	0.068
	0.700	0.102	0.091	0.096	0.098		0.700	0.065	0.062	0.063	0.064
	0.800	0.089	0.092	0.086	0.086		0.800	0.056	0.062	0.056	0.056
	0.900	0.067	0.091	0.069	0.067		0.900	0.042	0.062	0.043	0.042
	0.990	0.022	0.091	0.040	0.032		0.990	0.014	0.062	0.022	0.017
	0.999	0.007	0.091	0.025	0.025		0.999	0.004	0.062	0.012	0.011

Table 6: RMSE of estimators of binomial probability

n	p	PC(ELF,MLE)	PC(ELF,SLF)	PC(ELF,ASM)
10	0.001	0.010	0.990	0.990
	0.010	0.087	0.914	0.905
	0.100	0.465	0.536	0.246
	0.200	0.472	0.506	0.633
	0.300	0.476	0.512	0.419
	0.400	0.545	0.473	0.473
	0.500	0.584	0.352	0.577
	0.600	0.502	0.473	0.473
	0.700	0.487	0.511	0.419
	0.800	0.473	0.507	0.632
	0.900	0.465	0.536	0.247
	0.990	0.087	0.913	0.905
	0.999	0.010	0.991	0.990
20	0.001	0.019	0.980	0.980
	0.010	0.152	0.850	0.820
	0.100	0.481	0.688	0.579
	0.200	0.486	0.447	0.450
	0.300	0.488	0.501	0.495
	0.400	0.514	0.494	0.523
	0.500	0.578	0.390	0.520
	0.600	0.501	0.493	0.524
	0.700	0.487	0.500	0.495
	0.800	0.486	0.448	0.450
	0.900	0.480	0.688	0.579
	0.990	0.152	0.851	0.821
	0.999	0.019	0.980	0.980
50	0.001	0.047	1.000	0.951
	0.010	0.610	0.991	0.267
	0.100	0.495	0.623	0.485
	0.200	0.493	0.547	0.483
	0.300	0.495	0.474	0.508
	0.400	0.510	0.476	0.491
	0.500	0.523	0.473	0.520
	0.600	0.507	0.475	0.491
	0.700	0.496	0.475	0.508
	0.800	0.492	0.548	0.483
	0.900	0.495	0.623	0.486
	0.990	0.611	0.991	0.268
	0.999	0.046	1.000	0.951

Table 7: PC of estimators of binomial probability

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