fully indecomposable matrix A v is equal to the positive eigenvector then $A = \lambda DPD^{-1}$ where $D = \mathrm{diag}\,(v_1, \ldots, v_n)$, P is doubly stochastic and λ is the eigenvalue. If additionally $v = \mathbf{1}$ then A has to be a multiple of a doubly stochastic matrix. Diagonal matrices preserve the Hilbert volume so that D_2AD_1 contracts the volume least at the ray passing through $\mathbf{1}$ and also $\mathbf{1}$ is an eigenvector of D_2AD_1 with the eigenvalue equal to 1. Hence D_2AD_1 must be doubly stochastic.

For a fully indecomposable nonnegative $A = LPR$ we obtain from (10) and (13)

$$r(A) \geq (\det LR)^{1/n} \tag{14}$$

Moreover we have equality in (14) if and only if $r(A) = \rho(A)$. For a fully indecomposable $A \in \mathcal{P}$ it means that A contracts the Hilbert volume least at the ray passing through an eigenvector i.e. $L = \lambda R^{-1}$, $\lambda > 0$. (14) was first noted in [Lond] and from the proof there it follows immediately that as long as A is fully indecomposable we have equality in (14) if and only if $L = \lambda R^{-1}$ (without the nondegeneracy condition $A \in \mathcal{P}$). It follows also from [Lond] that (14) is valid whenever A allows the factorization $A = LPR$ where L, R are diagonal with positive diagonals and P is doubly stochastic (A not necessarily fully indecomposable).

The analogue of (14) for the maximal Lyapunov exponent can be formulated in the following way:

<u>Theorem 2</u> Let A_1, A_2, ... be a stationary, ergodic process with values in $n \times n$ non-negative matrices which allow the factorization $A_1 = LPR$, where L, R are diagonal matrices with positive diagonals and P is a doubly stochastic matrix, then the maximal Lyapunov exponent

$$\mu_{max} \geq \frac{1}{n}E(\ln \det LR)$$

<u>Proof</u> The inequality follows from (11) and (13) at least when A_1 are fully indecomposable. Generally $\rho(A_1) \geq \rho(L)\rho(P)\rho(R) = \det(LR)^{1/n}\rho(P)$ and $\rho(P) = 1$ for an arbitrary doubly stochastic matrix P (not necessarily fully indecomposable) which follows from [Lond]. $\square$

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