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A heuristic for sea-freight container selection, cargo allocation and cargo orientation

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A model is proposed to generate solutions for container selection, for the allocation of cargo to containers, and for cargo orientation within a container. The model is in the form of a mixed integer program with the objective of minimizing the total shipping cost. The practical requirements of loading priority and weight distribution along the main dimension of the container are incorporated into the model. A heuristic solution strategy is proposed and used to control the computation time by pre-setting the search increments. Three case examples are presented. The first and third examples show that the proposed model can produce a better solution than the manual schedulers. The second example is taken from the literature and is compared with the solution generated by the proposed model, demonstrating that the practical considerations incorporated into the model do not necessarily lead to increased shipping costs.

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Introduction

Shipment consolidation is the process of grouping different shipments from suppliers into a large shipment, which may involve several containers, see Tyan *et al* (2002). The aim of consolidation is to lower transportation costs through better utilization of the shipping containers' capacities. This problem occurs widely in many logistics operations, especially in transportation via rail, sea or air. There are numerous critical decisions to be made that affect costs, two of the most critical being the selection of the containers and the positioning of the cargos inside the selected containers. Although Hong Kong's container ports are currently the busiest in the world, such decisions are still being made by human operators, and, while the performance of experienced operators is generally high, work overload and unusual cargo mixes can often lead to poor and inconsistent performance.

Freight forwarders have three important decisions to make:

- decide the number and types of containers required, that is, the container selection plan;
- decide which items of cargo should be allocated to which container, that is, the cargo to the container allocation plan;
- decide the orientation of cargo items within a container, that is, the cargo position plan.

Considerable research has been conducted on developing mathematical models in which the main objective is volume utilization under the physical loading constraints (see Bischoff and Marriott, 1990; Bischoff and Ratcliff, 1995; Chen *et al*, 1995; Chien and Wu, 1998; Davies and Bischoff, 1999; Terno *et al*, 2000; Eley, 2002; He and Cha, 2002; Pisinger, 2002; Birgin *et al*, 2005). The work of Chen *et al* (1995) seems to form a good starting point for solving the problem when multiple containers, multiple carton sizes, carton orientations and possible overlapping of cartons in a container are all involved. However, the implementation of this work presents difficulties, not only because of the fact that the objective is restricted to container utilization but also because the model does not consider multiple containers, and the selection of containers can have a considerable impact on shipping costs. Xue and Lai (1997) and Mongeau and Bes (2003) include container selection in their models, but such models were developed for the air-freight industry and are not directly applicable to sea freight. Much of the research seems to be based on pure knapsack-type formulations, which tend to ignore many practical considerations. The wider problem involving both the selection of containers and the allocation of cargo for sea freight does not seem to have received much attention.

This paper proposes a model to generate plans for container selection, allocation of cargo to containers and for cargo orientation within a container. The model developed takes into consideration a number of practical requirements for sea-freight transportation. The problem is first modelled as a mixed integer program, which in theory can be solved to obtain the optimal container selection plan

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together with the optimal cargo allocation plan that minimizes the total shipping costs. However, computation time becomes prohibitively high even for relatively small examples and, therefore, a heuristic solution strategy that produces good suboptimal solutions must be adopted. The heuristic developed in this paper is a procedure that alternatively checks for the feasibility of the position of an item of cargo using a subset of co-ordinate points, instead of checking over all possible co-ordinate points. This enables the number of search loops to be controlled and the computing time to be made manageable.

Background and practices in cargo consolidation

Shipping lines offer different types of cargo containers of various dimensions, varying in total volume and weight that can be carried. The problem is further complicated by the fact that the ocean freight costs depend on several factors, including the shipping line charge, the container haulage charge and the terminal-handling charge. Also, there is usually more than one lot of cargo in a shipment and forwarders need to load cargo lots in the correct position so as to facilitate loading and unloading operations. A cargo lot refers to the items from the same shipper, and shippers can be located over a general catchment area. As the size and weight of the cargo items may not be the same, it is important to generate a feasible loading plan that places the cargo in stable positions inside containers. There are six important practical requirements, namely container weight, container size, stability, weight distribution, loading priority, orientation and handling. Container specifications issued by carriers will provide maximum payload information. Table 1 shows a selection of typical container specifications for a number of shipping lines; see also http://www.apl.com, http://www.oocl.com, http://www.maersksealand.com, and http://www.evergreen-marine.com.

When the cargo items inside a single container belong to more than one shipper, it is necessary to place the cargo items according to the pick-up sequence, that is, the sequence in which the container visits the shippers. If, for example, there are three lots of cargo from different shippers to be placed in one container, the first lot of cargo will be picked up from the first shipper and loaded in the head-load position. The container will then visit the second shipper and the second lot of cargo will be placed in the mid-load position. The third shipper will be visited last and the third lot of cargo will be placed in the tail-load position.

The terms ‘head load’, ‘mid load’ and ‘tail load’ simply refer to the loading order. The concept of loading priority is necessary to facilitate both the loading and unloading process when one container is used for more than one lot of cargo. In the case of more than one container, head load is defined as a first priority load, mid load refers to medium priority and tail load refers to the lowest priority. In the case when two containers are used to ship three lots of cargo from different shippers, containers will be filled sequentially: the first lot of cargo will be in the head-load position of the first container. If there is still enough space in the first container, it will visit the second shipper and the second lot of cargo will be placed in the mid-load position, but if there is insufficient space in the first container, the second lot of cargo will be

Table 1 Container specifications and ocean freight charges in example 1

Container number	*Shipping lines*	*Container type*	*Length (cm)*	*Width (cm)*	*Height (cm)*	*Maximum payload (kg)*	*Ocean freight charge* (HK$)*
1	*APL*	20′ Steel	590	235	239	21 850	7500
2	*OOCL*	20′ Steel	590	235	239	21 720	7500
3	*Evergreen*	20′ Steel	590	235	239	24 846	8500
4	*Maersk*	20′ Steel	590	235	239	24 850	8000
5	*APL*	40′ Steel	1203	235	239	26 760	16 600
6	*Evergreen*	40′ Steel	1203	235	239	26 480	16 550
7	*APL*	40′ Aluminium	1206	235	239	27 610	17 000
8	*Maersk*	40′ Aluminium	1206	235	238	29 710	18 000
9	*APL*	40′ HQ Steel	1203	235	270	26 750	17 600
10	*OOCL*	40′ HQ Steel	1203	235	270	26 500	17 550
11	*Evergreen*	40′ HQ Steel	1203	235	269	26 280	17 550
12	*Maersk*	40′ HQ Steel	1204	235	270	30 195	18 100
13	*APL*	45′ HQ Steel	1356	235	270	28 480	23 000
14	*APL*	45′ HQ Aluminium	1358	235	270	29 140	23 300

*The ocean freight charge refers to the export of normal cargo from Hong Kong to Rotterdam, which may vary from different forwarders.

placed in the head-load position of the second container. The third shipper will be visited last and the third lot of cargo will either be placed in the tail-load position of the first container or in the tail-load position of the second container.

Some cargo items must be placed in certain orientations and/or are required to be handled with special care. For example, if the cargo item is labeled 'This Way Up', it should be orientated in an acceptable direction. If an 'Easy To Break' notice is included, no cargo items are allowed to be placed on the top of that item, thus reducing the load pressure.

Weight distribution within a container also affects the stability of the container. It is desirable that its centre of gravity be as close as possible to the geometrical mid-point of the container floor. If the weight is very unevenly distributed, it may not be possible or it may be too difficult to carry out some container-handling operations. Also, in order to place the cargo items in a more stable manner, it may be necessary to lower the height of the centre of gravity, that is, lower the distance from the container floor. This means that heavier items should be placed nearer to the floor so as to lower the overall centre of gravity of the cargo items in the container. This requires the lighter cargo items to be placed on the top of the relatively heavier cargo items. It may even be necessary to restrict heavy items to placement below a certain height in the container. Our model considers the weight of the cargo items, but not the density distribution within a container. Shippers need only give cargo weight information to the shipping line for the generation of the bill of loading; the distribution of the density is not required.

Model and assumptions

The model proposed is in the class of bin-packing models, which in fact are NP-hard optimization problems. The work of Chen *et al* (1995) forms the main starting point for our model. The aim is to find the container selection plan, the cargo allocation plan and the cargo loading plan that minimizes the total shipping cost for a specified cargo set. Feasible solutions need to satisfy restrictions on container weight, container size, weight distribution and loading priority. Issues of stability, orientation and handling are not considered. Constraints on weight distribution apply only along the height dimension of the container; weight distribution along the length and width of the container can be ignored for practical purposes. A plan is regarded as feasible if all the following requirements are fulfilled:

1. No overlapping of the cargo items in each selected container(s).
2. The dimensions and weight of the cargo items do not exceed the dimensions and weight limit of the container, respectively.
3. The loaded positions of the cargo items fulfill the load-type requirements of the cargo lots. A *load type* refers to the loading order in the cargo-loading process and a *lot of cargo* refers to the cargo items that correspond to the same shipper, as explained above.
4. Light cargo items are on the top of the heavier cargo items. That means, the weight of the top cargo items must not exceed that of the lower cargo items.

The longest dimension of a cargo item is referred to as its length; the shortest dimension is the height; the middle one is the width. The container door opening direction is along its length (see Figure 1), which corresponds to the positive y-direction, the x-axis corresponds to the width and z-axis corresponds to the height. With respect to the relative positions of cargo items, $x_{ij}<x_{kj}$, $x_{ij}>x_{kj}$, $y_{ij}<y_{kj}$, $y_{ij}>y_{kj}$, $z_{ij}<z_{kj}$ and $z_{ij}>z_{kj}$ indicate that cargo item i is on the left-hand side of cargo item k, cargo item i is on the right-hand side of cargo item k, cargo item i is behind cargo item k, cargo item i is in front of cargo item k, cargo item i is under cargo item k and that cargo item i is on top of cargo item k.

Container variables

N is the total number of containers available.

C_j is the cost of shipping container j, that is, the sum of the ocean freight charge, terminal-handling charge and transportation (haulage) charge.

R_j, P_j, Q_j and W_j are the height, width, length and weight limit of container j, respectively.

Cargo variables

M is the total number of cargo items to be allocated.

r_i, p_i, q_i and w_i are the height, width, length and weight of cargo item i, respectively.

δ_i is the loading priority of cargo item i as discussed above. For example, if a lot of cargo consists of five cargo items and this lot of cargo is the first to be picked up (head load), then δ_1, δ_2, δ_3, δ_4 and δ_5 are equal to 1. To take another example, if there are three lots of cargo in the container and the second lot of cargo has four cargo items (say items 6, 7, 8 and 9) which will form the second pick-up (mid-load), then δ_6, δ_7, δ_8 and δ_9 will all be equal to 2.

α_{ij} are a set of six binary variables, 0 or 1, that define the orientation of item i in container j as specified in Table 2.

a_{ikj}, b_{ikj}, c_{ikj}, d_{ikj}, e_{ikj} and f_{ikj} are binary variables and equal to 1 if cargo item i is on the left-hand side, right-hand side, in front of, behind, above or under cargo item k in container j, and are equal to 0 otherwise.

N is an arbitrarily large positive integer.

x_{ij} is a discrete integer variable that refers to the x-coordinate of the back-bottom-left-hand corner of cargo item i in container j.

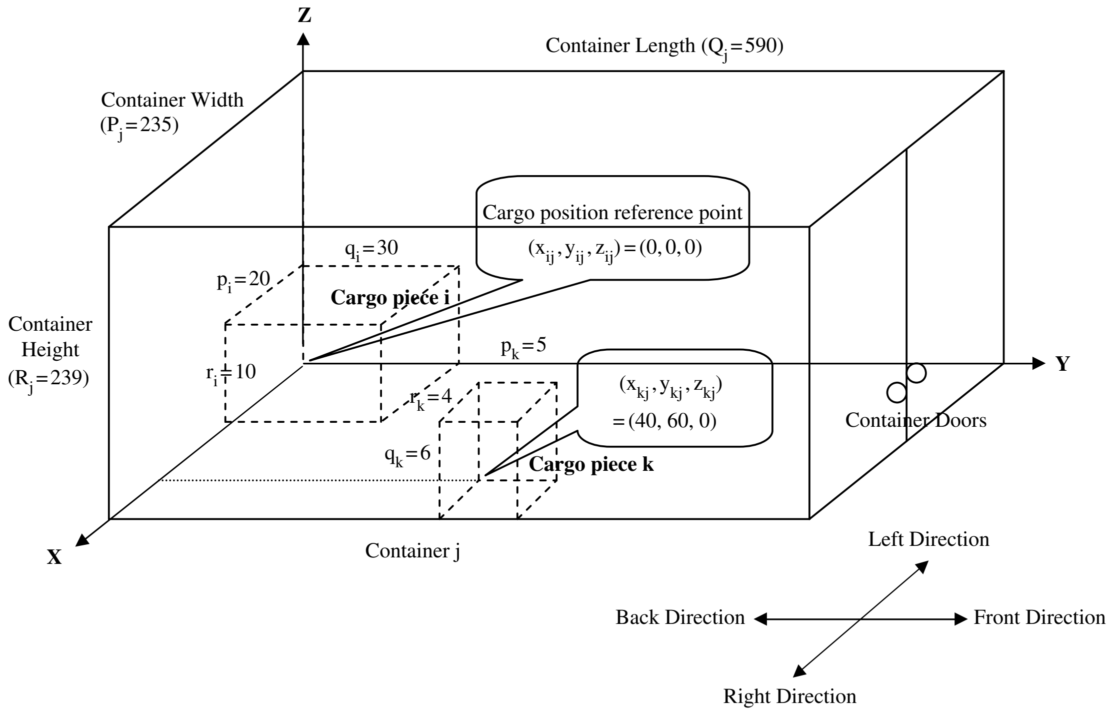

Figure 1 Illustration of cargo orientation and relative position.

Table 2 Cargo item orientation definition

	Parallel to x-axis	*Parallel to y-axis*	*Parallel to z-axis*
$\alpha_{a\ ij}=1$	Width	Length	Height
$\alpha_{b\ ij}=1$	Length	Width	Height
$\alpha_{c\ ij}=1$	Width	Height	Length
$\alpha_{d\ ij}=1$	Height	Width	Length
$\alpha_{e\ ij}=1$	Height	Length	Width
$\alpha_{f\ ij}=1$	Length	Height	Width

y_{ij} is a discrete integer variable that refers to the y-coordinate of the back-bottom-left-hand corner of cargo item i in container j.

z_{ij} is a discrete integer variable that refers to the z-coordinate of the back-bottom-left-hand corner of cargo item i in container j.

A_{ij} is a binary variable which equals 1 if cargo item i is assigned container number j and 0 if not.

With reference to Figure 1, $x_{ij}=0$; $y_{ij}=0$; $z_{ij}=0$ because the reference position of cargo item i in container j is at the origin. For cargo item k, the reference position (x_{kj}, y_{kj}, z_{kj}) is (40, 60, 0). Since $y_{ij}<y_{kj}$ and $x_{ij}<x_{kj}$, cargo item i is place behind and on the left-hand side of cargo item k. Thus $d_{ikj}=1$, $a_{ikj}=1$, and b_{ikj}, c_{ikj}, e_{ikj}, $f_{ikj}=0$. Regarding the cargo item orientation, the length, the height and the width of cargo item i are parallel to the y-, z- and x-axis, respectively, and the length, height and width of the cargo item k are parallel to the z-, x- and y-axis, respectively. So, $\alpha_{aij}=1$, $\alpha_{dkj}=1$ and α_{bij}, α_{cij}, α_{dij}, α_{eij}, α_{fij}, α_{akj}, α_{bkj}, α_{ckj}, α_{ekj}, $\alpha_{fkj}=0$.

The objective is to minimize the total shipping cost, that is:

$$\text{Min} \sum_{i=1}^{m}\sum_{j=1}^{n}\left[A_{ij}\left(C_j/\sum_{i=1}^{m}A_{ij}\right)\right]$$

subject to the following constraints:

$$\sum_{j=1}^{n} A_{ij} = 1 \quad \text{for all } i = 1, \ldots, m \tag{1}$$

$$1/\sum_{i=1}^{m} A_{ij} = 1 \quad \text{for } \sum_{i=1}^{m} A_{ij} = 0 \tag{2}$$

$$\begin{aligned} &x_{ij} + p_i(\alpha_{a\,ij} + \alpha_{c\,ij}) + q_i(\alpha_{b\,ij} + \alpha_{f\,ij}) + r_i(\alpha_{d\,ij} + \alpha_{e\,ij}) \\ &\quad \leqslant x_{kj} + N(1 - a_{ikj}) + N(1 - A_{ij}) + N(1 - A_{kj}) \\ &\quad \text{for all } i, j, k, i<k \end{aligned} \tag{3}$$

$$\begin{aligned} &x_{kj} + p_k(\alpha_{a\,kj} + \alpha_{c\,kj}) + q_k(\alpha_{b\,kj} + \alpha_{f\,kj}) + r_k(\alpha_{d\,kj} + \alpha_{e\,kj}) \\ &\quad \leqslant x_{ij} + N(1 - b_{ikj}) + N(1 - A_{ij}) + N(1 - A_{kj}) \\ &\quad \text{for all } i, j, k, i<k \end{aligned} \tag{4}$$

$$y_{kj} + p_k(\alpha_{b\,kj} + \alpha_{d\,kj}) + q_k(\alpha_{a\,kj} + \alpha_{e\,kj}) + r_k(\alpha_{c\,kj} + \alpha_{f\,kj}) \leqslant y_{ij} + N(1 - c_{ikj}) + N(1 - A_{ij}) + N(1 - A_{kj}) \quad \text{for all } i, j, k, i<k \tag{5}$$

$$y_{ij} + p_i(\alpha_{b\,ij} + \alpha_{d\,ij}) + q_i(\alpha_{a\,ij} + \alpha_{e\,ij}) + r_i(\alpha_{c\,ij} + \alpha_{f\,ij}) \leqslant y_{kj} + N(1 - d_{ikj}) + N(1 - A_{ij}) + N(1 - A_{kj}) \quad \text{for all } i, j, k, i<k \tag{6}$$

$$z_{kj} + p_k(\alpha_{e\,kj} + \alpha_{f\,kj}) + q_k(\alpha_{c\,kj} + \alpha_{d\,kj}) + r_k(\alpha_{a\,kj} + \alpha_{b\,kj}) \leqslant z_{ij} + N(1 - e_{ikj}) + N(1 - A_{ij}) + N(1 - A_{kj}) \quad \text{for all } i, j, k, i<k \tag{7}$$

$$z_{ij} + p_i(\alpha_{e\,ij} + \alpha_{f\,ij}) + q_i(\alpha_{c\,ij} + \alpha_{d\,ij}) + r_i(\alpha_{a\,ij} + \alpha_{b\,ij}) \leqslant z_{kj} + N(1 - f_{ikj}) + N(1 - A_{ij}) + N(1 - A_{kj}) \quad \text{for all } i, j, k, i<k \tag{8}$$

$$x_{ij} + p_i(\alpha_{a\,ij} + \alpha_{c\,ij}) + q_i(\alpha_{b\,ij} + \alpha_{f\,ij}) + r_i(\alpha_{d\,ij} + \alpha_{e\,ij}) \leqslant P_j + N(1 - A_{ij}) \quad \text{for all } i, j \tag{9}$$

$$y_{ij} + p_i(\alpha_{b\,ij} + \alpha_{d\,ij}) + q_i(\alpha_{a\,ij} + \alpha_{e\,ij}) + r_i(\alpha_{c\,ij} + \alpha_{f\,ij}) \leqslant Q_j + N(1 - A_{ij}) \quad \text{for all } i, j \tag{10}$$

$$z_{ij} + p_i(\alpha_{e\,ij} + \alpha_{f\,ij}) + q_i(\alpha_{c\,ij} + \alpha_{d\,ij}) + r_i(\alpha_{a\,ij} + \alpha_{b\,ij}) \leqslant R_j + N(1 - A_{ij}) \quad \text{for all } i, j \tag{11}$$

$$\sum_{i=1}^{m} w_i A_{ij} \leqslant W_j \quad \text{for all } j = 1, \ldots, n \tag{12}$$

$$N(1 - A_{ij}) + N(1 - A_{kj}) + N(1 - f_{ikj}) + N|(a_{ikj} - b_{ikj})| + N|(c_{ikj} - d_{ikj})| + w_i \geqslant w_k \quad \text{for all } i, j, k, i<k \tag{13}$$

$$N(1 - A_{ij}) + N(1 - A_{kj}) + N(1 - d_{ikj})(1 - f_{ikj}) + \delta_k \geqslant \delta_i \quad \text{for all } i, j, k, i<k \tag{14}$$

$$N(1 - A_{ij}) + N(1 - A_{kj}) + N(1 - c_{ikj})(1 - e_{ikj}) + \delta_i \geqslant \delta_k \quad \text{for all } i, j, k, i<k \tag{15}$$

$$\alpha_{a\,ij} + \alpha_{b\,ij} + \alpha_{c\,ij} + \alpha_{d\,ij} + \alpha_{e\,ij} + \alpha_{f\,ij} = 1 \quad \text{for all } i, j \tag{16}$$

$$a_{ikj} + b_{ikj} \leqslant 1 \quad \text{for all } i, j, k, i<k \tag{17}$$

$$c_{ikj} + d_{ikj} \leqslant 1 \quad \text{for all } i, j, k, i<k \tag{18}$$

$$e_{ikj} + f_{ikj} \leqslant 1 \quad \text{for all } i, j, k, i<k \tag{19}$$

$$0 \leqslant x_{ij} < P_j \quad \text{for all } i, j \tag{20}$$

$$0 \leqslant y_{ij} < Q_j \quad \text{for all } i, j \tag{21}$$

$$0 \leqslant z_{ij} < R_j \quad \text{for all } i, j \tag{22}$$

$$x_{ij}, y_{ij}, z_{ij}, \delta_i, s_j, C_j \geqslant 0 \quad \text{for all } i, j \tag{23}$$

$$A_{ij}, \alpha_{a\,ij}, \alpha_{b\,ij}, \alpha_{c\,ij}, \alpha_{d\,ij}, \alpha_{e\,ij}, \alpha_{f\,ij}, a_{ikj}, b_{ikj}, c_{ikj}, d_{ikj}, e_{ikj}, f_{ikj} = 0, 1 \quad \text{for all } i, j, k \tag{24}$$

Constraint (1) ensures that every cargo item is assigned to one and only one container. Constraint (2) ensures that the shipping cost is zero when no cargo item is assigned to the container. Constraints (3)–(8) ensure that there is no overlapping of cargo items under the six orientation patterns of cargo items in the three-dimensional planes. Constraints (9)–(11) restrict the length, width and height of the cargo item to being no longer than the dimensions of the container. Constraint (12) ensures that there is no violation of the container's weight limits in the loaded container(s). Constraint (13) guarantees the light cargo item(s) is/are on the top of the relatively heavier cargo item(s). Constraints (14) and (15) ensure that the loaded cargo items are all in the load-type ordering. For example, the head-loaded cargo lots are placed in the most inner part of the container and the tail-loaded cargo lots are placed near the container's door area. There are six possible orientation patterns for each cargo item (see Table 2), and, obviously, each cargo item only takes one orientation pattern; constraint (16) ensures that this is adhered to. Regarding the relative position between cargo items, constraints (17)–(19) restrict cargo items to feasible relative positions. The three-dimensional co-ordinates of the cargo items must lie within the dimensions of the container; constraints (20)–(22) ensure this. Constraint (23) guarantees the cargo item's three-dimensional co-ordinates, loading priority, cost factor and shipping cost to be positive integers. Constraint (24) ensures that the other variables are in zero–one status.

Since the dimension of the height of the cargo item cannot be zero, the feasible x, y and z values of the three-dimensional co-ordinates of the cargo items cannot be greater than the dimension of the container sides minus the dimension of the height of the cargo item. With reference to Figure 1, considering the container j and cargo item i, the maximum x, y and z values for its co-ordinates are, respectively, 235 minus 10, 590 minus 10 and 239 minus 10. Thus, no matter what the orientation of item i is, the feasible co-ordinates of item i will range from (0, 0, 0) to (225, 580, 229). Constraints (20)–(22) can therefore be modified as:

$$0 \leqslant x_{ij} \leqslant P_j - r_i \quad \text{for all } i, j \tag{20'}$$

$$0 \leqslant y_{ij} \leqslant Q_j - r_i \quad \text{for all } i, j \tag{21'}$$

$$0 \leqslant z_{ij} \leqslant R_j - r_i \quad \text{for all } i, j \tag{22'}$$

Proposed heuristic solution

The proposed procedure involves checking for feasibility in an incremental manner, instead of checking over all the possible co-ordinates. Three parameters are added to the model: the increment along the x-axis, ξ_x; the increment along the y-axis, ξ_y, and the increment along the z-axis, ξ_z. With the introduction of these increment values, the computing time becomes controllable, as the increment size determines the maximum number of search loops.

The algorithm uses the tree-search method to generate a set of possible co-ordinates for each cargo item in each container selection plan. It then uses a direct-substitution method to check whether the solution satisfies all the constraints. If all constraints are satisfied, the sets of co-ordinates are regarded as feasible and the shipping cost is calculated. After calculating the shipping costs of the feasible plans, the minimum cost plan is selected and the container utilization rates are calculated.

The main steps involved in the heuristic are as follows.

Step 1. Generates an initial cargo allocation plan, which includes information on which cargo items are allocated to which containers. Suppose there are four cargo items and two containers are available; it will generate a cargo allocation plan with parameters: $A_{11}=A_{21}=A_{31}=A_{41}=1$ and $A_{12}=A_{22}=A_{32}=A_{42}=0$. As A_{ij} is limited to 0–1 in this example, there are $4\times 2=8$ possible cargo allocation plans.

Step 2. For each cargo allocation plan the total shipping cost is calculated, that is, the sum of ocean freight charge, container haulage charge and terminal-handling charge for the selected container(s). If, for example, $A_{11}=A_{21}=A_{31}=A_{41}=1$ and $A_{12}=A_{22}=A_{32}=A_{42}=0$, then only container 1 is selected and the total shipping cost involves container 1 only.

Step 3. Compares the shipping costs incurred for the cargo allocation plan with the lowest shipping cost generated so far. The initial shipping cost is set to an arbitrarily large positive integer, and if shipping cost is greater than the lowest shipping cost generated so far it will go to Step 11.

Step 4. Generate a possible cargo-loading plan. The cargo-loading plan refers to the exact three-dimensional co-ordinates of the cargo items in the containers. For example, $x_1=1$, $y_1=2$, $z_1=3$ dictates that the co-ordinates of the cargo item 1 are (1, 2, 3).

Step 5. This step examines constraints (3)–(8) in the model, and checks if there is any overlapping of the cargo items in the selected container(s). It begins by checking the orientations of the cargo items in the x-, y- and z-direction, and then checks if there is any overlapping of cargo items in x-, y- and z-direction. If there is overlapping in any direction, then the plan is infeasible and the algorithm jumps to Step 10.

Step 6. This step examines constraints (9)–(11) of the model, checking if the dimensions of the cargo items are greater than those of the container. It begins by checking the orientations of the cargo items and then generating information on the dimensions of the cargo items in x-, y- and z-direction. It then compares the dimensions of the cargo items with those of the container. If the dimensions of the cargo items are greater than those of the container, the plan is infeasible and algorithm jumps to Step 10.

Step 7. This step examines constraint (12) of the model. It checks if the total weight of all the cargo items is greater than the container weight limit. If the total weight of all the cargo in a container is greater than the container weight limit, the plan is infeasible and the algorithm jumps to Step 10.

Step 8. This step examines constraint (13) of the model by checking if light cargo items are on the top of the relatively heavier cargo items. If the weight of a cargo item is greater than that of an item directly underneath it, the plan is infeasible and algorithm jumps to Step 10.

Step 9. This step examines constraints (14) and (15) of the model. It checks if all the cargo items are loaded according to the load-type requirement. It compares the loading priority of any two cargo items in the container, and if the cargo item, say item i, is behind or/and under the other item, say item k ($d_{ikj}=1$ or/and $f_{ikj}=1$). If the loading priority of item k is smaller than that of item i, then the cargo-loading plan is regarded as infeasible. It also checks if cargo item i is in front of and/or above item k ($c_{ikj}=e_{ikj}=1$), and, if the loading priority of item i is smaller than that of item k, the cargo-loading plan will then be also regarded as infeasible. If the plan is infeasible, it proceed to Step 10, otherwise it goes to Step 11.

Step 10. Add an increment in x-direction and then proceed to Step 4. This is done until x reaches the limit, $P_j - r_i$. When this limit is reached, an increment will be added to the y- and z-direction alternatively. When all the co-ordinate points for all cargo items have been considered, it proceeds to Step 11; otherwise it goes back to Step 4.

Step 11. The allocation and loading plan with the lowest shipping cost is selected and its total shipping cost and container utilization rates are computed.

Examples and discussion

The heuristic was coded using the Java programming language, enabling flexibility through the use of web-based information processing. This has practical advantages when running the system on a regular basis and the input and output can be tailored to the needs of the industry.

Example 1

The application of the model is illustrated by using data from a Hong Kong freight-forwarding company. The shipping orders involve 15 three-dimensional cargo items that need to be shipped from Hong Kong to Rotterdam using normal container(s). Contractual agreements between the shipping line and the forwarder give the forwarder 14 container types to select from. The container specifications and freight charges of the 14 container types are shown in Table 1; the cargo item specification and load type are shown in Table 3.

This case problem with 14 containers has 16 343 ($2^{14}-1$) possible container combinations, a large number even for such a relatively small problem. The manual schedulers, by considering the total volume and approximate costs, are able to limit their search to a few container types. This problem is further complicated by the size and weight limitations of each container type. The total sea-freight shipping cost is the sum of the ocean freight charge, terminal-handling charge and local transportation charge. These charges will vary depending on the container type (see Tables 1 and 4). In most cases, the total shipping cost calculation is the summation of ocean freight, terminal-handling charge and local transportation charge. But for some special loads, it may also include others such as dangerous goods surcharge, war risk surcharge, fuel surcharge, document-handling charge, etc.

Table 3 Cargo item specification and load type in example 1

Cargo item number	*Cargo dimensions (cm)*	*Load type*	*Cargo weight (kg)*
1	150 × 200 × 40	1	510
2	196 × 420 × 71	1	993
3	150 × 217 × 103	2	498
4	107 × 204 × 50	2	2988
5	172 × 204 × 103	2	1003
6	90 × 210 × 15	2	1000
7	100 × 200 × 90	2	500
8	120 × 150 × 110	3	4580
9	125 × 169 × 115	3	5040
10	200 × 200 × 100	3	3096
11	111 × 212 × 98	4	504
12	192 × 219 × 85	4	1060
13	106 × 126 × 75	5	3111
14	128 × 130 × 102	5	540
15	209 × 212 × 156	6	4244

The data include the cargo and container specifications, the container-shipping costs and the load types of the cargo items. The solution generated using pre-set increment values of $\xi_x=20$, $\xi_y=20$ and $\xi_z=20$ is shown in Table 5. The values of A_{ij} give information on the container selection and cargo allocation. The cargo-loading position is given by the respective co-ordinates, x_{ij}, y_{ij}, and z_{ij}. The orientation of each cargo item in the container is given by $\alpha_{a\ ij}$, $\alpha_{b\ ij}$, $\alpha_{c\ ij}$, $\alpha_{d\ ij}$, $\alpha_{e\ ij}$, and $\alpha_{f\ ij}$. The results show that the forwarder should select container numbers 1 and 4, leading to a total shipping cost of HK$22 230 and a container utilization rate of 61.97%.

Experience with this type of example, which is typical of virtually all small to medium freight forwarders, suggests that it is very unlikely that a plan that achieves a container utilization rate above 90% can be found. This is because of the irregular shapes and the different load types of each of the cargo items. In actual practice, the container utilization rate will range from 60 to 80%, depending on seasonal factors. The forwarders, of course, have to deliver the container(s) to the port even when the container utilization rate is less than 60% and container utilization rates between 55 and 65% are not uncommon.

Comparing the model's result with those from the forwarder's operators, it was found that they had selected container number 8, leading to a total shipping cost of HK$18 000 + HK$2750 + HK$2000 = HK$22 750. This suggests that the saving resulting from using the proposed model is around 2.29%. This figure can be expected to increase as the number of cargo items increases.

The operators do not seem to have any formal approach to the loading problem. They use their loading experience with a limited number of trial-and-error attempts. This can lead to a heavy work load when the number of orders is large, and they may need to off-load some cargo items and procure another container when they discover that their loading plan is in fact infeasible. This usually leads to

Table 4 Local haulage charges and terminal-handling charges of cargo container

Container types	*Local container haulage charges (HK$) (Cost may vary for different forwarders)*	*Terminal-handling charges (HK$) (Export container from Hong Kong to European countries)*
20′ Steel/ aluminium	1300	2065
40′ Steel/ aluminium	2000	2750
40′ High-cube steel/aluminium	2200	2750
45′ High-cube steel/aluminium	2500	2750

Table 5 Result for example 1

Cargo item number 1	*Container assignment*														*Coordinates of cargo item*			*Cargo item orientation pattern*					
	A_{i1}	A_{i2}	A_{i3}	A_{i4}	A_{i5}	A_{i6}	A_{i7}	A_{i8}	A_{i9}	A_{i10}	A_{i11}	A_{i12}	A_{i13}	A_{i14}	x_{ij}	y_{ij}	z_{ij}	$\alpha_{a\ ij}$	$\alpha_{b\ ij}$	$\alpha_{c\ ij}$	$\alpha_{d\ ij}$	$\alpha_{e\ ij}$	$\alpha_{f\ ij}$
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	150	0	0	0	0	0	0	1	0
3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	40	0	0	0	0	1	0	0
4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	420	0	0	0	1	0	0	0
5	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	200	0	0	0	0	1	0	0
6	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	200	220	1	0	0	0	0	0
7	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	480	0	0	0	1	0	0	0
8	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	120	0	1	0	0	0	0	0
9	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
10	0	0	0	1	0	0	0	0	0	0	0	0	0	0	120	0	0	0	0	0	0	1	0
11	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	180	0	0	0	1	0	0	0
12	0	0	0	1	0	0	0	0	0	0	0	0	0	0	120	200	0	0	0	0	0	1	0
13	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	280	0	1	0	0	0	0	0
14	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	280	80	0	0	0	0	1	0
15	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	420	0	0	0	1	0	0	0
Average utilization rate for all selected container(s):																		61.97%					
Total shipping cost:																		HK$22 230					

increased shipping costs and loading time, whereas the proposed model will at the very least produce a feasible solution.

Although there are 14 container types available for the forwarder to choose from, only about five types of container (container numbers 1, 5, 7, 9 and 10 in Table 3) are generally used. This is done to reduce the complexity of the container selection process and is also possibly the results of user bias towards certain types of containers. Prior contractual arrangements between the forwarders and the shipping lines can also be one of the factors limiting the choice of containers.

It took 45 min to find the solution using an AMD 950 MHz computer running under the J2EE operating environment; the pre-set increments x, y and z were all 20. The resulting solution has a shipping cost of HK$22 230 and a container utilization rate of 61.97%. Experience suggests that the human operator will need more than 1 h to device a suitable plan for allocating the cargo to suitable container(s); however, there is no guarantee that the most economical allocation can be achieved, and it is also quite possible that the allocation plan proposed will at times be very poor.

While this example may be regarded as rather simple, in that it only involves one container type, it is not too dissimilar to the one used by Chen *et al* (1995), and it is intended to demonstrate the role played by the cargo-loading priority and cargo weight constraints.

Example 2

In order to better explore the effect of the cargo weight constraint and loading priority limitation on the loading process, a test case from Chen *et al* (1995) is to used compare their results with those of the proposed model. As Chen *et al*'s (1995) model does not consider container weight and loading priority limitations, their problem needs to be modified to include these two factors. The problem involves six cargo items and its specifications are shown in Table 6. In Chen *et al*'s (1995) model, the objective is to minimize the length of the container required to pack all the cargo, and thus only one container is involved (see Table 7

Table 6 Cargo item specification and load type for example 2

Cargo item number	*Cargo dimensions (in) (data extracted from Chen et al, 1995)*	*Load type (Head load/full load = '1')*	*Cargo weight (kg)*
1	6 × 25 × 8	3	540
2	5 × 20 × 10	1	3500
3	3 × 16 × 7	2	600
4	6 × 15 × 12	1	3100
5	3 × 22 × 8	2	330
6	4 × 20 × 10	2	2300

for specifications). Using the pre-set increments of $\xi_x = 1$, $\xi_y = 1$ and $\xi_z = 1$, the solution generated by the proposed model is shown in Table 8. The results show that cargo items 1, 3 and 5 should be loaded in the upper layer of the container, whereas items 2, 4 and 6 should be loaded in the lower layer of the container, resulting in a total shipping cost of HK\$5000 and a container utilization rate of 70.63%.

As the pre-set increments ξ_x, ξ_y and ξ_z were all one, the results generated from the proposed model and by Chen *et al*'s (1995) model are both similar (see Table 9). A comparison shows that the two sets of results differ in the cargo-loading positions only. Owing to the cargo weight constraint and the loading priority limitation in the proposed model, cargo item 1 is loaded in the outermost position of the container, since this item has tail-load priority, whereas items 2 and 4 are loaded in the innermost position of the container as these items have been assigned a head-loaded priority. Items 1, 3 and 5 are loaded in the upper layer of the container because light cargo items have to be placed on the top of the heavier cargo items for stability reasons.

A full comparison of the results generated from the proposed model with those using the model of Chen *et al* (1995) is inappropriate, because the proposed model addresses the practical issues of loading priority and weight distribution. For this example, the model of Chen *et al* (1995) contains 198 constraints and 153 variables, and it took 15 min to find the optimum solution using the LINGO package on a DEC5000/P200 computer. By comparison, our model took 22 min to find the solution presented using a pre-set increment of one for all three dimensions. Using the J2EE environment on an AMD 950 MHz computer, the computation time was also lower for Chen *et al*'s model. However, this is not unexpected since our model incorporated the additional constraints resulting from loading priority and weight distribution.

Table 7 Container specification and ocean freight charges for example 2

Container number	*Length (in)*	*Width (in)*	*Height (in)*	*Maximum payload (kg)*	*Ocean freight charge (HK\$)*
1	35	20	10	12 000	5000

Example 3

This (see Table 10) is a more practical and comprehensive example and shows the effect of load-type limitation on container selection and cargo allocation. The number of

Table 8 Result for example 2

Cargo item number i	*Container assignment*	*Coordinates of cargo item i*			*Cargo item orientation pattern*					
	A_{i1}	x_{i1}	y_{i1}	z_{i1}	$\alpha_{a\ i1}$	$\alpha_{b\ i1}$	$\alpha_{c\ i1}$	$\alpha_{d\ i1}$	$\alpha_{e\ i1}$	$\alpha_{f\ i1}$
1	1	7	12	4	0	1	0	0	0	0
2	1	0	0	0	1	0	0	0	0	0
3	1	0	0	5	0	1	0	0	0	0
4	1	7	0	1	0	1	0	0	0	0
5	1	20	0	4	0	1	0	0	0	0
6	1	7	10	0	0	1	0	0	0	0
Average utilization rate under all selected container(s):						70.63%				
Total shipping cost:						HK\$5000				

Table 9 Comparison of the computational test results between Chen *et al*'s (1995) model and the proposed model

Cargo item number i	*Coordinates of cargo item i (result generated from Chen et al's model)*			*Coordinates of cargo item (result generated from the proposed model)*		
	x_{i1}	x_{i1}	y_{i1}	z_{i1}	y_{i1}	z_{i1}
1	7	12	4	7	12	4
2	0	0	5	0	0	0
3	0	0	0	0	0	5
4	20	0	4	7	0	1
5	7	0	1	20	0	4
6	7	10	0	7	10	0

container types available has been increased to 19 (see Table 1 and its extension, Table 11). The costs involved are summarized in Table 12. The fact that most of the cargo dimensions were divisible by 5 was used to reduce the number of search loops and therefore the computation time by pre-setting the increment values (ξ_x, ξ_y and ξ_z) to 5. The CPU time required under the J2EE environment on an AMD 950 MHz computer was 65 min. The solution (see Table 13) recommends that one each of container types 1, 3 and 10 be used, leading to a total shipping cost of HK$45 230 and a container utilization rate of 74.81%. The solution indicates that cargo items 1, 2, 3, 7, 19 and 20, cargo items 12, 13, 14 and 18, cargo items 4, 5, 6, 8, 9, 10, 11, 15, 16 and 17 should be loaded in container types 1, 3 and 10, respectively.

Table 10 Cargo item specification and load type in example 3

Cargo item number	*Cargo dimensions (cm)*	*Load type*	*Cargo weight (kg)*
1	135 × 400 × 120	1	1200
2	115 × 380 × 95	1	3300
3	100 × 200 × 100	1	4000
4	170 × 330 × 110	2	2200
5	100 × 620 × 22	2	1500
6	180 × 220 × 100	2	2000
7	120 × 240 × 104	3	3200
8	200 × 500 × 120	4	1800
9	80 × 600 × 20	4	1200
10	170 × 420 × 150	4	700
11	70 × 550 × 18	4	1200
12	80 × 220 × 40	5	1000
13	200 × 230 × 200	5	12 000
14	180 × 210 × 85	5	2500
15	180 × 380 × 100	6	300
16	200 × 250 × 150	7	6500
17	232 × 610 × 30	7	8800
18	220 × 290 × 215	8	8000
19	150 × 170 × 140	8	2600
20	200 × 345 × 90	8	5500

When compared with the results of the operator's manual solution, it was found that they had selected container numbers 1, 13 and 15, leading to a total shipping cost of HK$10 865 + HK$10 865 + HK$28 250 = HK$49 980. These savings resulting from using the proposed model are around 9.50%, a figure consistent with our experience in applying our model to problems with similar configurations.

Concluding comments

The task of cargo consolidation and container loading is a challenging one because there are a number of decisions to be made and many practical issues to be considered. Generally speaking, the problem of cargo consolidation can be broken down into three subproblems. The first subproblem is the decision on the selection and renting of which types of container and how many containers from which shipping lines. The second subproblem is the allocation of which cargo items to which containers. The third subproblem is the exact loading co-ordinates of the cargo items in the selected container(s), which satisfy the constraints of the practical requirements in the loading process. The proposed model takes into account a number of

Table 12 Local haulage charges and terminal-handling charges of cargo container

Container types	*Local container haulage charges (HK$) (cost may vary for different forwarders)*	*Terminal-handling charges (HK$) (export container from Hong Kong to European countries)*
20′ Steel/ aluminium	1300	2065
40′ Steel/ aluminium	2000	2750
40′ High-cube steel/aluminium	2200	2750
45′ High-cube steel/aluminium	2500	2750

Table 11 Container specifications and ocean freight charges (extension to Table 1)

Container number	*Shipping lines*	*Container type*	*Length (cm)*	*Width (cm)*	*Height (cm)*	*Maximum payload (kg)*	*Ocean freight charge (HK$)*
15	*APL*	20′ Steel	590	235	239	21 850	7500
16	*OOCL*	20′ Steel	590	235	239	21 720	7500
17	*Evergreen*	20′ Steel	590	235	239	24 846	8500
18	*Maersk*	20′ Steel	590	235	239	24 850	8000

Table 13 Result for example 3

	Container assignment																		*Coordinates of cargo item i*			*Cargo item orientation pattern*					
Cargo item number i	A_{i1}	A_{i2}	A_{i3}	A_{i4}	A_{i5}	A_{i6}	A_{i7}	A_{i8}	A_{i9}	A_{i10}	A_{i11}	A_{i12}	A_{i13}	A_{i14}	A_{i15}	A_{i16}	A_{i17}	A_{i18}	x_{ij}	y_{ij}	z_{ij}	$\alpha_{a\ ij}$	$\alpha_{b\ ij}$	$\alpha_{c\ ij}$	$\alpha_{d\ ij}$	$\alpha_{e\ ij}$	$\alpha_{f\ ij}$
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100	0	95	0	0	0	0	1	0
2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100	0	0	1	0	0	0	0	0
3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	220	0	1	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	180	0	0	0	0	0	0	1	0
6	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
7	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100	0	0	0	0	1	0
8	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	180	1	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	200	0	170	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	30	550	0	0	0	0	0	1	0
11	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	200	0	100	0	0	0	0	1	0
12	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	200	180	0	1	0	0	0	0
13	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
14	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	200	0	0	0	0	0	0	1
15	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	30	550	170	1	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	30	970	0	0	0	1	0	0	0
17	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	500	0	0	0	0	0	1	0
18	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	285	0	0	0	0	0	1	0
19	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	90	400	0	0	0	0	1	0	0
20	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	240	0	0	0	0	0	1	0

Average utilization rate for all selected container(s): 74.81%

Total shipping cost: HK$45 230

practical requirements of the sea-freight transportation industry.

The proposed model is novel in the sense that it has incorporated the two practical issues of loading priority and weight distribution. There is also a degree of novelty in the use of fixed increments in the three dimensions to control the computation time. Another novel element is that the total shipping cost is considered. However, there are many other practical considerations that can be included in developing the model further. The assumption that the shapes of cargo items are three-dimensional boxes is generally acceptable, because shippers usually pack their cargos into pallets or into boxes before containerization. The assumption of an unlimited number of containers being available is also generally acceptable because there is usually more than one shipping line covering a particular shipping route. So if a shipping line is not able to provide the required number of containers, the forwarder can simply ask other shipping lines.

An obvious direction for further research is to better define the weight distribution problem; our model has included one dimension only, that is, along the height of the container. The weight distribution along the length and width of the container were ignored, but may well be of considerable importance in other packing problems. Another significant extension of the model could be the inclusion of loading priority into the optimization model. In the proposed model, the loading priority (pick up sequence in the catchment area) of each cargo item is given, but this in itself can form an interesting travelling salesman problem. Fixing the search increments dynamically could also be an interesting area for further research. Here the software would need to internally change the increments dynamically, depending on the number of orders and elapsed CPU time. The computational loads that these problems generate are extremely heavy, but fortunately the cost of CPU time has become an insignificant factor. Finally, it is important that such heuristics be packaged as web-based systems, tailored to the customary input/output formats used in the industry, and not just simply presented as integer programming software packages.

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